

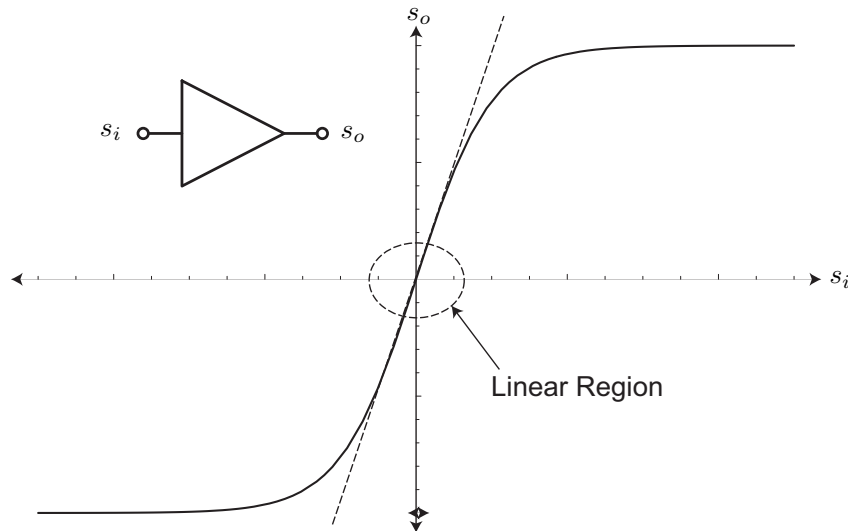
Lecture 10: Distortion Analysis

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Introduction to Distortion



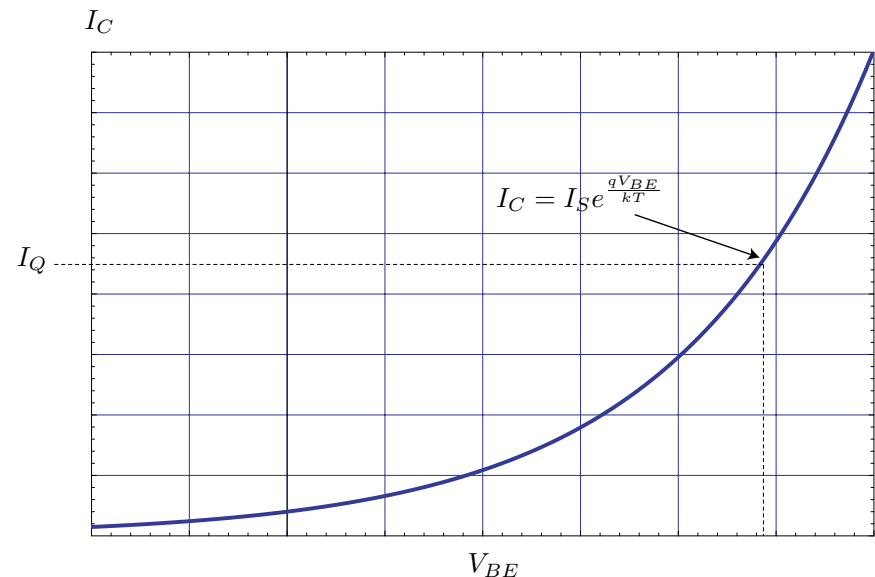
- Up to now we have treated amplifiers as small-signal linear circuits. Since transistors are non-linear, this assumption is only valid for extremely small signals.
- Consider a class of memoryless non-linear amplifiers. In other words, let's neglect energy storage elements.
- This is the same as saying the output is an instantaneous function of the input. Thus the amplifier has no memory.

Distortion Analysis Assumptions

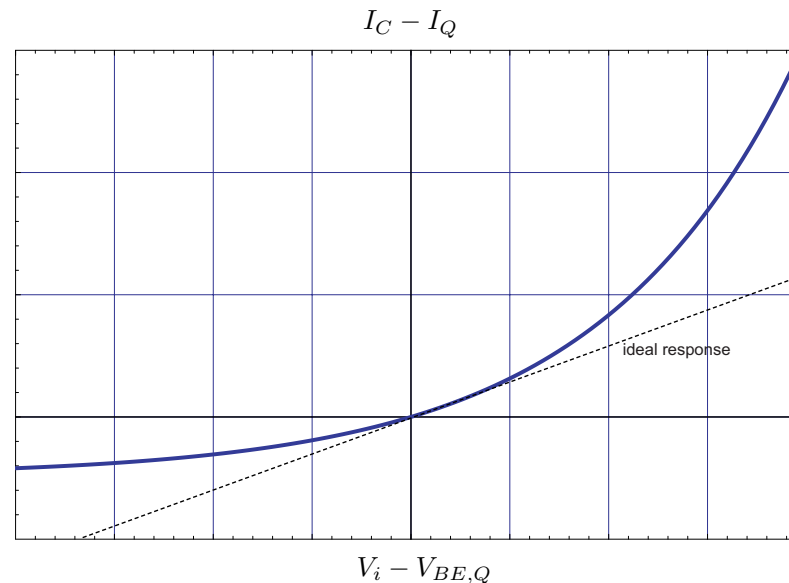
- We also assume the input/output description is sufficiently smooth and continuous as to be accurately described by a power series

$$s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \dots$$

For instance, for a BJT (Si, SiGe, GaAs) operated in forward-active region, the collector current is a smooth function of the voltage V_{BE}

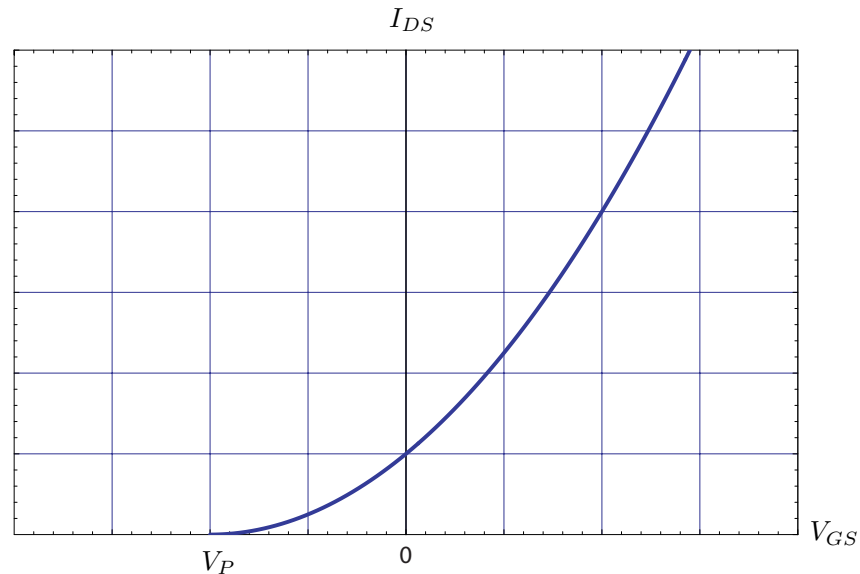


BJT Distortion



- We shift the origin by eliminating the DC signals, $i_o = I_C - I_Q$. The input signal is then applied around the DC level $V_{BE,Q}$.
- Note that an ideal amplifier has a perfectly linear line.

JFET Distortion

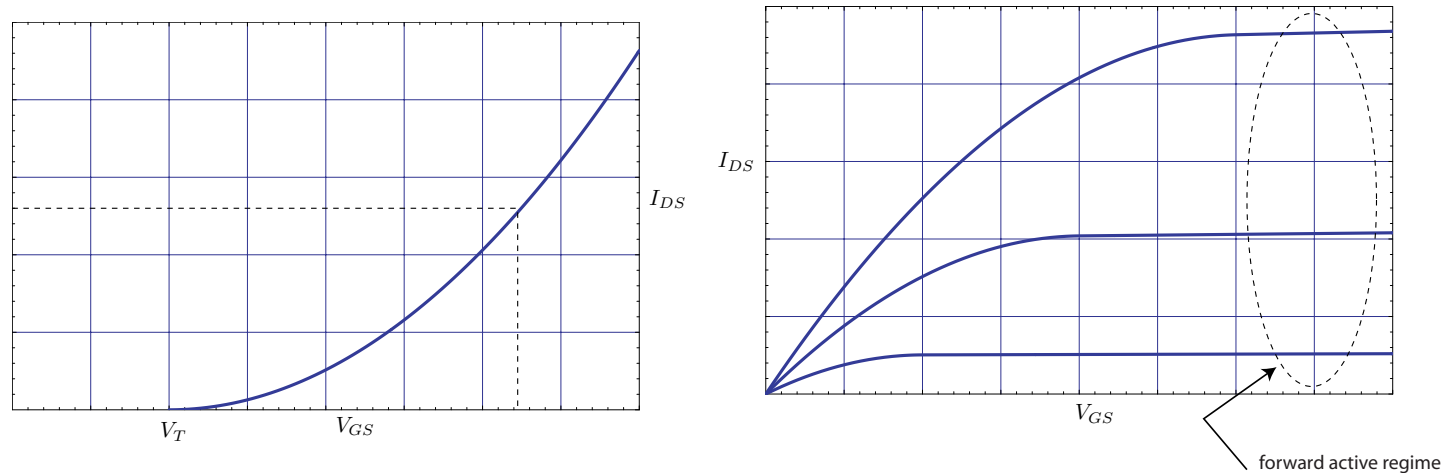


- JFETs are more common devices in RF circuits. The I-V relation is also approximately square law

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

- The gate current (junction leakage) is typically very small $I_G \sim 10^{12}$ A. So for all practical purposes, $R_i = \infty$.

MOSFET Distortion



- The long-channel device also follows the square law relation (neglecting bulk charge effects)

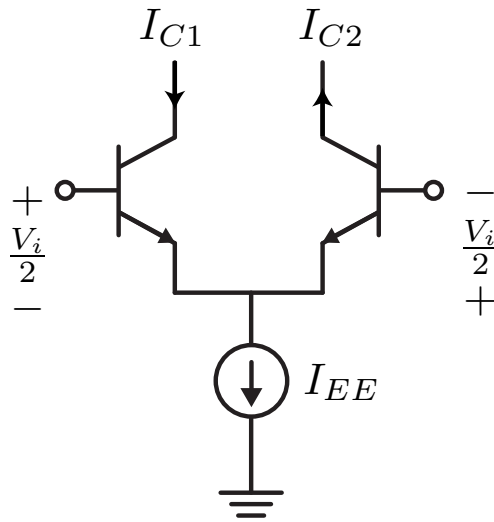
$$I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$$

- This is assuming the device does not leave the forward active (saturation) regime.

MOSFET Model

- Note that the device operation near threshold is not captured by our simple square-law equation
- The I-V curve of a MOSFET in moderate and weak inversion is easy to describe in a “piece-meal” fashion, but difficult to capture with a single equation.
- Short-channel devices are even more difficult due to velocity saturation and drain induced barrier lowering.

Differential Pair



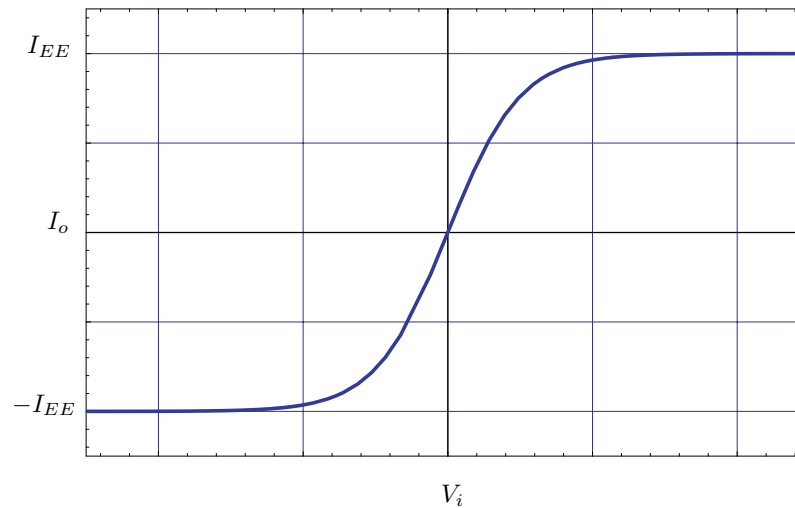
The differential pair is an important analog and RF building block.

- For a BJT diff pair, we have $V_i = V_{BE1} - V_{BE2}$

$$I_{C1,2} = I_S e^{\frac{qV_{BE1,2}}{kT}}$$

- The sum of the collector currents are equal to the current source $I_{C1} + I_{C2} = I_{EE}$

BJT Diff Pair



- The ideal BJT diff pair I-V relationship (neglecting base and emitter resistance) is given by

$$I_o = I_{C1} - I_{C2} = \alpha I_{EE} \tanh \frac{qV_i}{2kT}$$

- Notice that the output current saturates for large input voltages

Power Series Relation

- For a general circuit, let's represent this behavior with a power series

$$s_o = a_1 s_i + a_2 s_i^2 + a_3 s_i^3 + \dots$$

- a_1 is the small signal gain
- The coefficients a_1, a_2, a_3, \dots are independent of the input signal s_i *but* they depend on bias, temperature, and other factors.

Harmonic Distortion

- Assume we drive the amplifier with a time harmonic signal at frequency ω_1

$$s_i = S_1 \cos \omega_1 t$$

- A linear amplifier would output $s_o = a_1 S_1 \cos \omega_1 t$ whereas our amplifier generates

$$s_o = a_1 S_1 \cos \omega_1 t + a_2 S_1^2 \cos^2 \omega_1 t + a_3 S_1^3 \cos^3 \omega_1 t + \dots$$

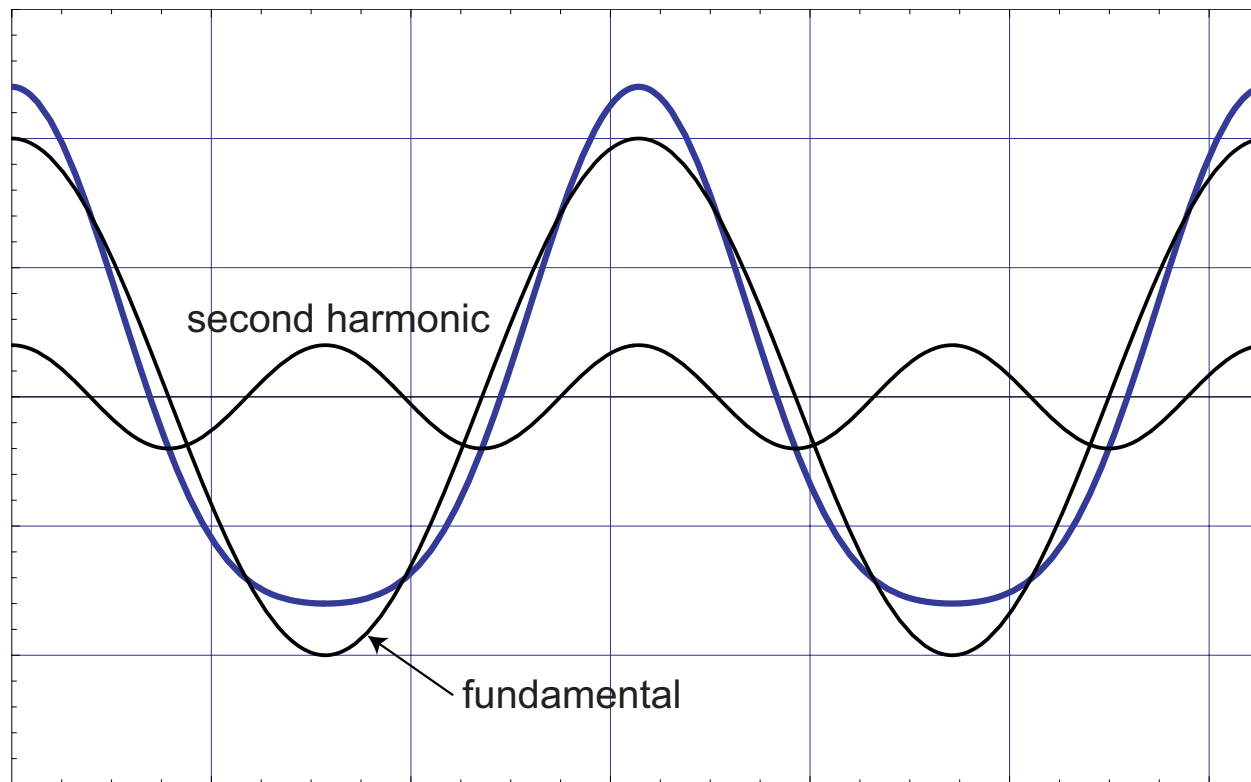
or

$$s_o = a_1 S_1 \cos \omega_1 t + \frac{a_2 S_1^2}{2} (1 + \cos 2\omega_1 t) + \frac{a_3 S_1^3}{4} (\cos 3\omega_1 t + 3 \cos \omega_1 t) + \dots$$

Harmonic Distortion (cont)

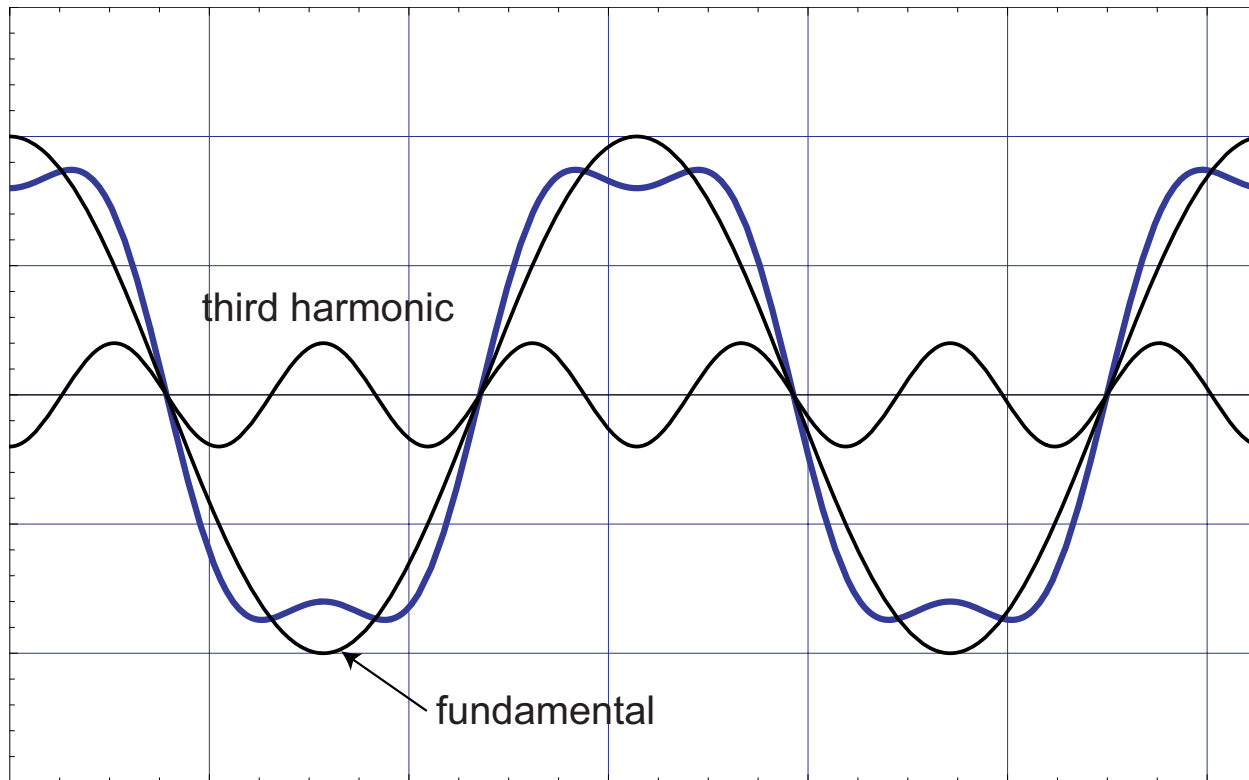
- The term $a_1 s_1 \cos \omega_1 t$ is the wanted signal.
- Higher harmonics are also generated. These are unwanted and thus called “distortion” terms. We already see that the second-harmonic $\cos 2\omega_1 t$ and third harmonic $\cos 3\omega_1 t$ are generated.
- Also the second order non-linearity produces a DC shift of $\frac{1}{2} a_2 S_1^2$.
- The third order generates both third order distortion and more fundamental. The sign of a_1 and a_3 determine whether the distortion product $a_3 S_1^3 \frac{3}{4} \cos \omega_1 t$ adds or subtracts from the fundamental.
- If the signal adds, we say there is gain expansion. If it subtracts, we say there is gain compression.

Second Harmonic Disto Waveforms



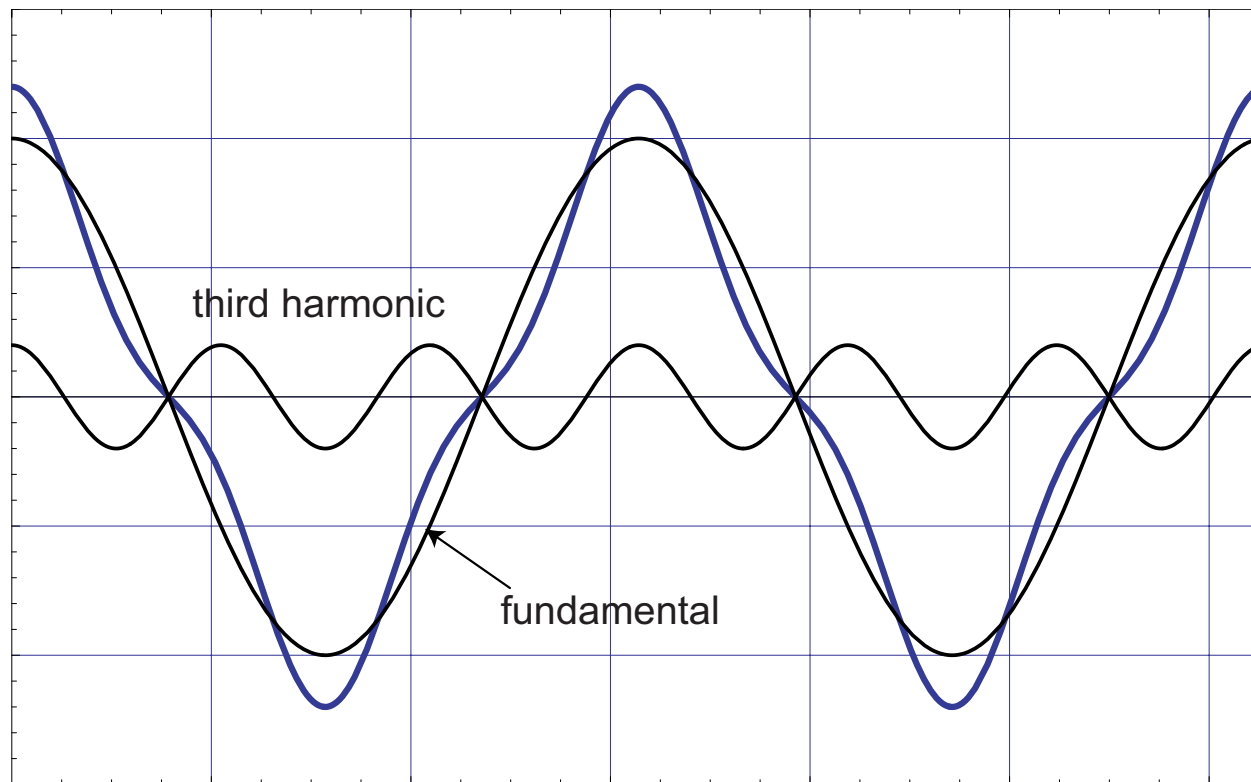
- The figure above demonstrates the waveform distortion due to second harmonic only.

Third Harmonic Distortion Waveform



- The above figure shows the effects of the third harmonic, where we assume the third harmonic is in phase with the fundamental.

Third Harmonic Waveform (cont)



- The above figure shows the effects of the third harmonic, where we assume the third harmonic is out of phase with the fundamental.

General Distortion Term

- Consider the term $\cos^n \theta = \frac{1}{2^n} (e^{j\theta} + e^{-j\theta})^n$. Using the Binomial formula, we can expand to

$$= \frac{1}{2^n} \sum_{k=0}^n \binom{n}{k} e^{jk\theta} e^{-j(n-k)\theta}$$

- For $n = 3$

$$\begin{aligned} &= \frac{1}{8} \left(\binom{3}{0} e^{-j3\theta} + \binom{3}{1} e^{j\theta} e^{-j2\theta} + \binom{3}{2} e^{j2\theta} e^{-j\theta} + \binom{3}{3} e^{j3\theta} \right) \\ &= \frac{1}{8} (e^{-j3\theta} + e^{j3\theta}) + \frac{1}{8} 3 (e^{j\theta} + e^{-j\theta}) = \frac{1}{4} \cos 3\theta + \frac{3}{4} \cos \theta \end{aligned}$$

General Distortion Term (cont)

- We can already see that for an odd power, we will see a nice pairing up of positive and negative powers of exponentials
- For the even case, the middle term is the unpaired DC term

$$\binom{2k}{k} e^{jk\theta} e^{-jk\theta} = \binom{2k}{k}$$

- So only even powers in the transfer function can shift the DC operation point.
- The general term in the binomial expansion of $(x + x^{-1})^n$ is given by

$$\binom{n}{k} x^{n-k} x^{-k} = \binom{n}{k} x^{n-2k}$$

General Distortion Term (cont)

- The term $\binom{n}{k} x^{n-2k}$ generates every other harmonic.
- If n is even, then only even harmonics are generated. If n is odd, likewise, only odd harmonics are generated.
- Recall that an “odd” function $f(-x) = -f(x)$ (anti-symmetric) has an odd power series expansion

$$f(x) = a_1x + a_3x^3 + a_5x^5 + \dots$$

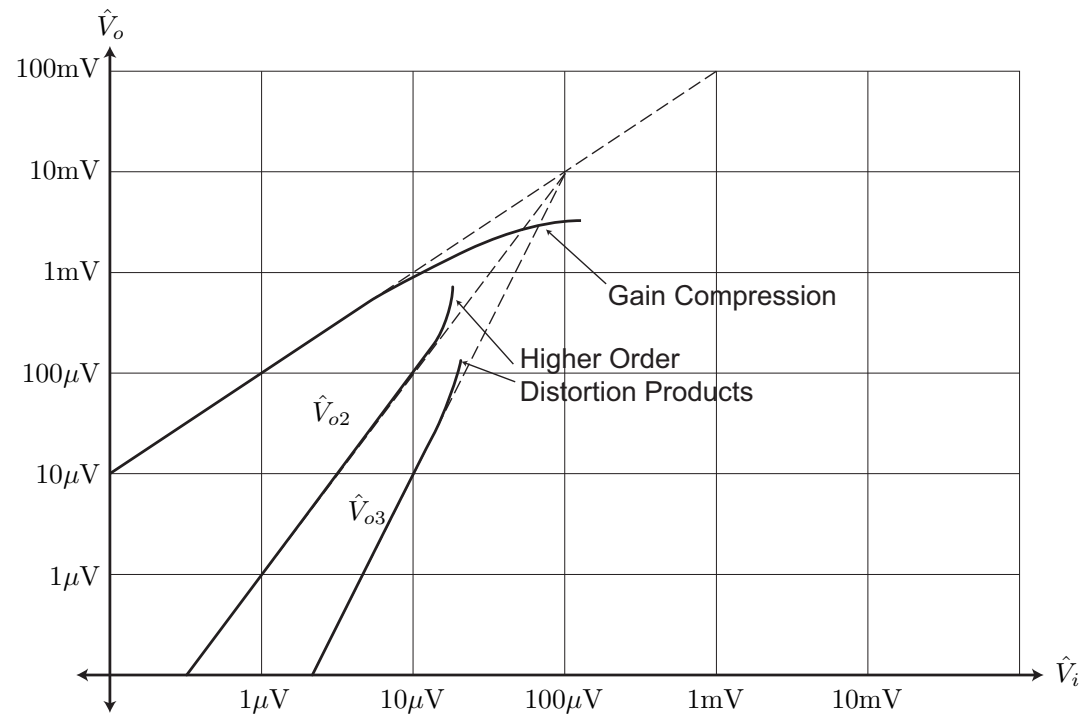
- Whereas an even function, $g(-x) = g(x)$, has an even power series expansion

$$g(x) = a_0 + a_2x^2 + a_4x^4 + \dots$$

Output Waveform

- In general, then, the output waveform is a Fourier series

$$v_o = \hat{V}_{o1} \cos \omega_1 t + \hat{V}_{o2} \cos 2\omega_1 t + \hat{V}_{o3} \cos 3\omega_1 t + \dots$$



Fractional Harmonic Distortion

- The fractional second-harmonic distortion is a commonly cited metric

$$HD_2 = \frac{\text{ampl of second harmonic}}{\text{ampl of fund}}$$

- If we assume that the square power dominates the second-harmonic

$$HD_2 = \frac{a_2 \frac{S_1^2}{2}}{a_1 S_1}$$

or

$$HD_2 = \frac{1}{2} \frac{a_2}{a_1} S_1$$

Third Harmonic Distortion

- The fractional third harmonic distortion is given by

$$HD_3 = \frac{\text{ampl of third harmonic}}{\text{ampl of fund}}$$

- If we assume that the cubic power dominates the third harmonic

$$HD_3 = \frac{a_3 \frac{S_1^2}{4}}{a_1 S_1}$$

or

$$HD_3 = \frac{1}{4} \frac{a_3}{a_1} S_1^2$$

Output Referred Harmonic Distortion

- In terms of the output signal S_{om} , if we again neglect gain expansion/compression, we have $S_{om} = a_1 S_1$

$$HD_2 = \frac{1}{2} \frac{a_2}{a_1^2} S_{om}$$

$$HD_3 = \frac{1}{4} \frac{a_3}{a_1^3} S_{om}^2$$

- On a dB scale, the second harmonic increases linearly with a slope of one in terms of the output power whereas the third harmonic increases with a slope of 2.

Signal Power

- Recall that a general memoryless non-linear system will produce an output that can be written in the following form

$$v_o(t) = \hat{V}_{o1} \cos \omega_1 t + \hat{V}_{o2} \cos 2\omega_1 t + \hat{V}_{o3} \cos 3\omega_1 t + \dots$$

- By Parseval's theorem, we know the total power in the signal is related to the power in the harmonics

$$\begin{aligned} \int_T v^2(t) dt &= \int_T \sum_j \hat{V}_{oj} \cos(j\omega_1 t) \sum_k \hat{V}_{ok} \cos(k\omega_1 t) dt \\ &= \sum_j \sum_k \int_T \hat{V}_{oj} \cos(j\omega_1 t) \hat{V}_{ok} \cos(k\omega_1 t) dt \end{aligned}$$

Power in Distortion

- By the orthogonality of the harmonics, we obtain Parseval's Theorem

$$\int_T v^2(t) dt = \sum_j \sum_k \frac{1}{2} \delta_{jk} \hat{V}_{oj} \hat{V}_{ok} = \frac{1}{2} \sum_j \hat{V}_{oj}^2$$

- The power in the distortion relative to the fundamental power is therefore given by

$$\frac{\text{Power in Distortion}}{\text{Power in Fundamental}} = \frac{V_{o2}^2}{V_{o1}^2} + \frac{V_{o3}^2}{V_{o1}^2} + \dots$$
$$= HD_2^2 + HD_3^2 + HD_4^2 + \dots$$

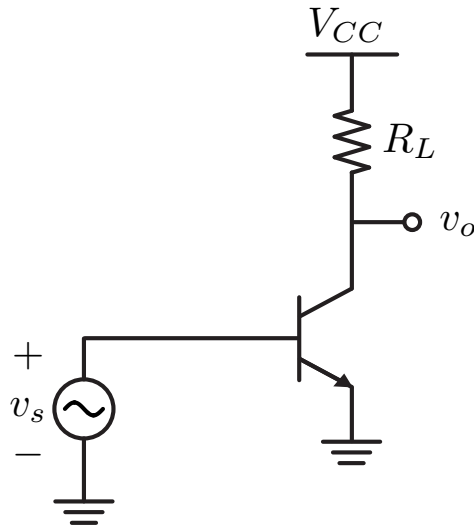
Total Harmonic Distortion

- We define the *Total Harmonic Distortion (THD)* by the following expression

$$THD = \sqrt{HD_2^2 + HD_3^2 + \dots}$$

- Based on the particular application, we specify the maximum tolerable *THD*
- Telephone audio can be pretty distorted ($THD < 10\%$)
- High quality audio is very sensitive ($THD < 1\%$ to $THD < .001\%$)
- Video is also pretty forgiving, $THD < 5\%$ for most applications
- Analog Repeaters $< .001\%$. RF Amplifiers $< 0.1\%$

Distortion of BJT Amplifiers



- Consider the CE BJT amplifier shown. The biasing is omitted for clarity.

- The output voltage is simply

$$V_o = V_{CC} - I_C R_C$$

- Therefore the distortion is generated by I_C alone. Recall that

$$I_C = I_S e^{qV_{BE}/kT}$$

BJT CE Distortion (cont)

- Now assume the input $V_{BE} = v_i + V_Q$, where V_Q is the bias point. The current is therefore given by

$$I_C = \underbrace{I_S e^{\frac{V_Q}{V_T}}}_{I_Q} e^{\frac{v_i}{V_T}}$$

- Using a Taylor expansion for the exponential

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$$

$$I_C = I_Q \left(1 + \frac{v_i}{V_T} + \frac{1}{2} \left(\frac{v_i}{V_T} \right)^2 + \frac{1}{6} \left(\frac{v_i}{V_T} \right)^3 + \dots \right)$$

BJT CE Distortion (cont)

- Define the output signal $i_c = I_C - I_Q$

$$i_c = \frac{I_Q}{V_T} v_i + \frac{1}{2} \left(\frac{q}{kT} \right)^2 I_Q v_i^2 + \frac{1}{6} \left(\frac{q}{kT} \right)^3 I_Q v_i^3 + \dots$$

- Compare to $S_o = a_1 S_i + a_2 S_i^2 + a_3 S_i^3 + \dots$

$$a_1 = \frac{qI_Q}{kT} = g_m$$

$$a_2 = \frac{1}{2} \left(\frac{q}{kT} \right)^2 I_Q$$

$$a_3 = \frac{1}{6} \left(\frac{q}{kT} \right)^3 I_Q$$

Example: BJT HD2

- For any BJT (Si, SiGe, Ge, GaAs), we have the following result

$$HD_2 = \frac{1}{4} \frac{q\hat{v}_i}{kT}$$

- where \hat{v}_i is the peak value of the input sine voltage
- For $\hat{v}_i = 10\text{mV}$, $HD_2 = 0.1 = 10\%$
- We can also express the distortion as a function of the output current swing \hat{i}_c

$$HD_2 = \frac{1}{2} \frac{a_2}{a_1^2} S_{om} = \frac{1}{4} \frac{\hat{i}_c}{I_Q}$$

- For $\frac{\hat{i}_c}{I_Q} = 0.4$, $HD_2 = 10\%$

Example: BJT IM3

- Let's see the maximum allowed signal for $IM_3 \leq 1\%$

$$IM_3 = \frac{3}{4} \frac{a_3}{a_1} S_1^2 = \frac{1}{8} \left(\frac{q\hat{v}_i}{kT} \right)^2$$

- Solve $\hat{v}_i = 7.3\text{mV}$. That's a pretty small voltage. For practical applications we'd like to improve the linearity of this amplifier.