1. (25 points) A conductor with $\sigma = 10^7 \text{S/m}$ and dielectric constant $\varepsilon = \varepsilon_0$ is placed into a uniform electric field $E_0 = 50 \text{V/m}$.

(a) Find the charge density everywhere. Show the location of the charge distribution schematically on the figure above.

(b) Estimate the time scale to reach the steady-state charge distribution.

\[ E_0 \]

\[ \rightarrow \rightarrow \rightarrow \rightarrow \]

\[ \rightarrow \rightarrow \rightarrow \rightarrow \]

\[ \sigma \]

\[ \varepsilon_0 \]

\[ \rightarrow \rightarrow \rightarrow \rightarrow \]

\[
\begin{align*}
\text{INSIDE THE CONDUCTOR, } & E = 0, \ \varepsilon = 0 \\
\text{AT SURFACE, } & \varepsilon_s = \hat{n} \cdot \mathbf{D} = \varepsilon_0 E_0 \\
\varepsilon_s &= 50 \times 8.854 \times 10^{-12} \text{ C/m}^2 = 443 \text{ pC/m}^2
\end{align*}
\]

(b) Estimate the time scale to reach the steady-state charge distribution.

\[
\begin{align*}
\text{SEVERAL TIME CONSTANTS} \\
\tau &= \frac{\varepsilon_0}{\sigma} \approx 9 \times 10^{-19} \\
\text{THE EVOLUTION IS EXPONENTIAL}
\end{align*}
\]
(c) Is there a force on the conductor? If so, draw the direction of the force and calculate the magnitude.

\[
\text{NET FORCE IS ZERO}
\]

\[
-qE_0 \rightarrow qE_0
\]

\[
q = \oint A
\]

(d) Now a dielectric object is placed in the field as shown below. Half of the material has dielectric constant \( \varepsilon_1 = 2 \) while the other half has \( \varepsilon_2 = 4 \). Is there a force on the object? If so, calculate the magnitude and indicate the direction of the force.

\[
E_0
\]

\[
\text{YES, THE BOUND POLARIZATION CHARGE IS NOT THE SAME IN THE TWO REGIONS.}
\]

\[
\begin{align*}
\Sigma S_1 &= -\varepsilon_0 E_0 \left( 1 - \frac{\varepsilon_0}{\varepsilon_1} \right) = -\frac{1}{2} \varepsilon_0 E_0 \\
\Sigma S_2 &= +\varepsilon_0 E_0 \left( 1 - \frac{\varepsilon_0}{\varepsilon_2} \right) = \frac{3}{4} \varepsilon_0 E_0 \\
\Sigma S_3 &= -\left[ \varepsilon_0 E_0 \left( 1 - \frac{\varepsilon_0}{\varepsilon_1} \right) - \left( -\varepsilon_0 E_0 \left( 1 - \frac{\varepsilon_0}{\varepsilon_2} \right) \right) \right] \\
&= -\Sigma S_3 - \Sigma S_1 = -\frac{1}{4} \varepsilon_0 E_0
\end{align*}
\]

\[
\begin{align*}
F_1 &= -\frac{1}{2} \varepsilon_0 E_0^2 A \\
F_2 &= -\frac{3}{4} \varepsilon_0 E_0^2 A \\
F_3 &= \frac{3}{4} \varepsilon_0 E_0^2 A
\end{align*}
\]
3. (30 points) A transmission line is formed by running a single wire of radius $a$ parallel to earth. The cross-section of the transmission line is shown below. We wish to find the transmission line parameters. You may assume that the region above the wire is air and thus $\varepsilon \approx \varepsilon_0$, $\mu \approx \mu_0$.

(a) Find the potential in the charge-free region for $y > 0$. You may assume that the wire radius is negligible. Hint: Use the method of images.

\[
\phi = -\frac{8\varepsilon}{2\pi\varepsilon_0} \left( \ln \left| r - d \right| - \ln \left| r + d \right| \right)
\]

(b) Find the capacitance per unit length of the line.

\[
C_1 = \frac{8\varepsilon}{V_0} = \frac{2\pi\varepsilon_0}{2\ln \left( \frac{2d}{a} - 1 \right)}
\]
(c) Find the inductance per unit length of the line.

\[ L' = \frac{\mu_0}{2\pi} \ln \left| \frac{2d}{a} - 1 \right| \]

(d) Calculate the shunt conductive loss \( G' \) per unit length.

\[ RC = \frac{\varepsilon_0}{\sigma} \]

\[ G = \frac{C\sigma}{\varepsilon_0} = \frac{2\pi \sigma}{\ln \left| \frac{2d}{a} - 1 \right|} \]

(e) Estimate the series resistance per unit length \( R' \). You may assume that current flows uniformly in a layer of thickness \( \delta \) along the outer edges of the conductors. Assume earth has a conductivity of \( \sigma_2 \). Hint:

\[ \int_{-\infty}^{\infty} \frac{y}{x^2 + y^2} = \pi \]
(f) Calculate the line characteristic impedance.

\[ Z_0 = \sqrt{\frac{jZ_0 L' + R'}{jZ_0 C' + G'}} \]

**Current On Skin:**

\[ R = \frac{R_{wire} + R_{end}}{2} = \frac{1}{2\pi a}\]

**Tend = Send - N Dn**

\[ Send = 208 \]

\[ R_{wire} = \frac{1}{a} - \frac{1}{2\pi a} \]
\[
\int_{-\infty}^{\infty} \left( \frac{d}{\pi (x^2 + d^2)} \right)^2 dx = \frac{1}{2\pi d} = \frac{1}{W_{\text{eff}}}
\]

\[
R'_{\text{GND}} = \frac{1}{\sigma_{\text{GND}} 2\pi d s}
\]

Effective Area

\[
R_T' = R'_{\text{GND}} + R'_{\text{WIRE}} = \frac{1}{\sigma_{\text{GND}} 2\pi d s} + \frac{1}{\sigma 2\pi a s}
\]
4. (10 points) Using the fact that a capacitor stores energy equal to $\frac{1}{2}CV^2$, and the electrostatic field has energy density of $\frac{1}{2}E \cdot D$, calculate the capacitance of the following structure by equating the total energy stored in the cap to the energy contained in the electrostatic field. \textit{Neglect} fringing fields.

\begin{align*}
E_1 &= E_2 = \frac{V_0}{t} \quad D_1 = \varepsilon_1 E_1 \\
D_2 &= \varepsilon_2 E_2 \\
E &= \frac{1}{2} \int_{V_1} E \cdot D \, dV + \frac{1}{2} \int_{V_2} E \cdot D \, dV \\
&= \frac{1}{2} \left( \frac{w_1 \varepsilon_1 V_0^2}{t} \right) + \frac{1}{2} \left( \frac{(w-w_1) \varepsilon_2 V_0^2}{t} \right) \\
&= \frac{1}{2} C' V_0^2 \\
C' &= \frac{w_1 \varepsilon_1}{t} + \frac{(w-w_1) \varepsilon_2}{t}
\end{align*}