# Phase Noise in *LC* Oscillators

Konstantin A. Kouznetsov and Robert G. Meyer

***Abstract*—Analytical methods for the phase-noise analysis of *LC*-tuned oscillators are presented. The fundamental assumption used in the theoretical model is that an oscillator acts as a large-signal *LC*-tuned amplifier for purposes of noise analysis. This approach allows us to derive closed-form expressions for the close-to-carrier spectral density of the output noise, and to estimate the phase-noise performance of an oscillator from circuit parameters using hand analysis. The emphasis is on an engineering approach intended to facilitate rapid estimation of oscillator phase noise. Theoretical predictions are compared with results of circuit simulations using a nonlinear phase-noise simulator. The analytical results are in good agreement with simulations for weakly nonlinear oscillators. Complete nonlinear simulations are necessary to accurately predict phase noise in oscillators operating in a strongly nonlinear regime. To confirm the validity of the nonlinear phase-noise models implemented in the simulator, simulation results are compared with measurements of phase noise in a practical Colpitts oscillator, where we find good agreement between simulations and measurements.**

***Index Terms*—Jitter, noise, oscillator, phase noise, voltage-controlled oscillator.**

## I. INTRODUCTION

VOLTAGE-CONTROLLED oscillators (VCO's) are essential building blocks of wireless communication systems, where the spectral quality of the local oscillator signal directly influences the out-of-band interference. Phase noise in oscillators has long been the subject of theoretical and experimental investigation. An early model of phase noise, introduced by Leeson [1], qualitatively described phase-noise spectra in a variety of oscillators. Theoretical derivations of the Leeson formula rely mostly on a linear time-invariant approach to the analysis of noise in oscillators [2]–[4]. Treatments of nonlinear effects on oscillator noise have also been developed [5], [6]. Recently, general theories that allow quantitative predictions of phase noise have been introduced [7]–[9]. These exact approaches naturally lend themselves to computer-aided analysis. A complete nonlinear phase-noise simulator that utilizes approaches similar to those of [7]–[9] has been implemented in a commercial simulation package [10].

In this paper, we explore the validity and limitations of linear noise analysis of *LC*-tuned oscillators. We examine and exploit the assumption that an *LC* feedback oscillator acts as a high-gain, large-signal *LC* amplifier with respect to its noise sources despite the nonlinear limiting that occurs in steady-state oscillation. This approach allows us to make quantitative estimates of the phase-noise performance of *LC* oscillators with a weak nonlinearity, giving predictions which are in good agreement with nonlinear simulations with the SpectreRF simulator. Our engineering techniques provide insights and quantitative understanding of noise in oscillators and serve as a starting point in a design procedure for practical oscillators that should include complete nonlinear simulations.

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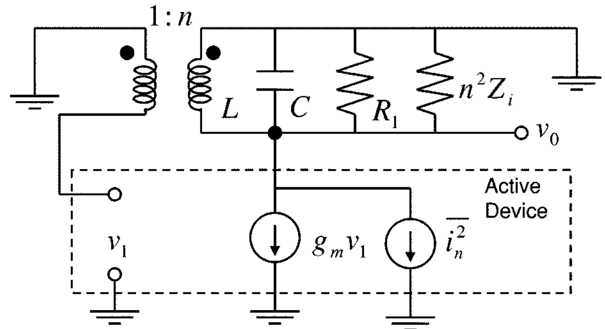


Fig. 1. AC schematic diagram of an *LC*-tuned transformer-coupled oscillator showing a single current noise source referred to the output of the active device.

## II. OSCILLATOR NOISE THEORY

In the analysis of phase noise of an *LC* oscillator, we assume that an oscillator is operating in the steady state at a fundamental frequency $f_0$ and only wide-sense stationary noise sources are superimposed onto the oscillating waveform. In a nonlinear system noise sidebands at frequencies $kf_0 \pm \delta f$ also contribute to output noise at close-to-carrier frequencies $f_0 \pm \delta f$ via processes of nonlinear mixing; here, $k$ is an integer between $+\infty$ and $-\infty$. For clarity of the discussion that follows, we derive the phase noise of the oscillator at the output frequency $f_0 - \delta f$. We further assume that the oscillator is operating in a near-linear fashion such that noise sidebands close to the carrier provide the dominant contribution to oscillator noise and mixing from other harmonics is suppressed.

Without loss of generality, the circuit diagram of an *LC*-tuned oscillator can be represented as shown in Fig. 1 [11]. The loop gain is assumed to be just less than unity, so that the circuit behaves as a high-gain *LC* positive-feedback amplifier. As the oscillation amplitude builds up, the influence of the negative resistance provided by the active device is reduced (due to soft limiting or AGC action) such that in the steady state, the output spectrum of an oscillator contains a very sharp noise peak at its fundamental frequency, resulting from the oscillator limiting on its own amplified noise.

To determine the noise transfer function from the noise generator $\overline{i_n^2}$ to the oscillator output, we consider effects of noise

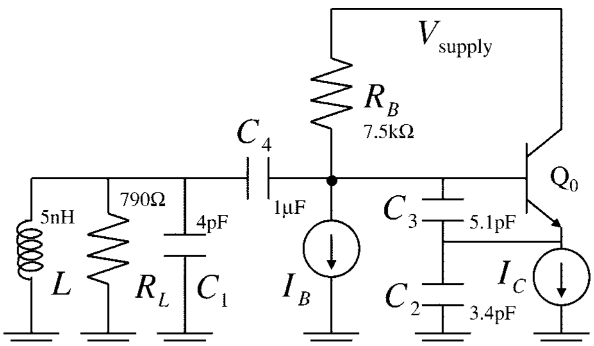


Fig. 2. Circuit diagram of a Colpitts oscillator with the collector of $Q_0$ ac grounded.

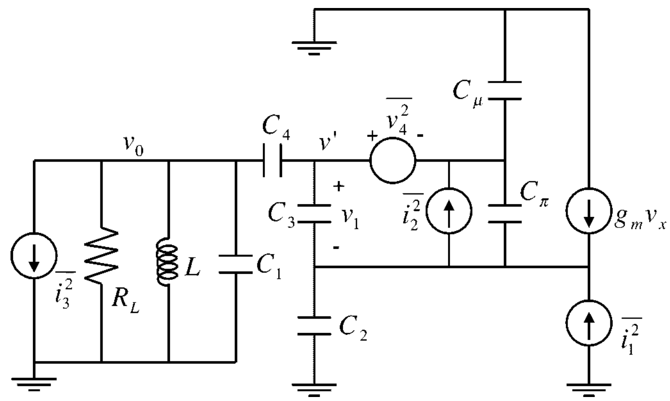


Fig. 3. Small-signal model of the Colpitts oscillator used for the phase noise analysis.

modulation to positive and negative sidebands. As we show in the Appendix, the noise power at $f_0 - \delta f$ will be equally distributed between sidebands at $f_0 - \delta f$ and $-f_0 - \delta f$ leading to the noise transfer function $\alpha_0$ at $f_0 - \delta f$, which can be written as

$$\alpha_0 = \frac{1}{2} \times \frac{R_L}{2jQ} \left(\frac{f_0}{\delta f}\right). \tag{1}$$

This expression explicitly contains a factor of 1/2 resulting from noise spreading to positive and negative frequencies. Similarly, a noise tone at $-f_0 - \delta f$, which is uncorrelated with the tone at $f_0 - \delta f$, is spread evenly between $-f_0 - \delta f$ and $f_0 - \delta f$ in the output noise spectrum. To determine the total noise at the output at $f_0 - \delta f$, contributions from positive and negative frequencies must be added in quadrature leading to the total noise spectral density at $f_0 - \delta f$, $\overline{v}_t^2$, given by

$$\overline{v}_t^2 = 2 \times \overline{i}_n^2 \times \frac{1}{4} \times \frac{R_L^2}{4Q^2} \left(\frac{f_0}{\delta f}\right)^2. \tag{2}$$

We note that the total output noise of an *LC* oscillator given by (2) is 1/2 of the noise power of an *LC* tank with the same $Q$. This distinction in the noise shaping between an oscillator and a resonant circuit has been pointed out previously in [10].

It is frequently stated that close-to-carrier oscillator noise can be decomposed into AM- and PM-modulated parts with the AM-modulated part being suppressed due to amplitude limiting in the steady state of oscillation. This reasoning often leads to the inclusion of a factor of 1/2 into phase-noise expressions. We perform our calculations in a manner which is similar to the noise analysis of a positive-feedback amplifier, and do not include an additional factor of 1/2 in derivations. Therefore, we use the terms oscillator noise and phase noise interchangeably, meaning total noise at the oscillator output.

## III. Colpitts Oscillator Analysis

To test the validity of our approach to the noise analysis of *LC* oscillators, we analyze phase noise in a Colpitts oscillator. A circuit diagram of a Colpitts oscillator with the collector of the active device ac grounded is shown in Fig. 2. The active device is a bipolar-junction transistor $Q_0$ biased by dc current sources $I_C$ and $I_B$, and resistor $R_B$. The capacitive transformer formed by capacitors $C_2$ and $C_3$ provides the positive feedback needed to induce oscillation. The coupling capacitor $C_4$ can be chosen to provide additional transformations from the base of $Q_0$ to the tank circuit, or a large value of $C_4$ can be chosen to provide a short circuit between the base of $Q_0$ and the tank at the oscillation frequency.

We consider the Colpitts oscillator biased at a relatively low value of the collector bias current, $I_C = 0.2$ mA. This value is chosen to set the initial loop gain at 1.5, which is sufficiently close to unity to create a weakly nonlinear oscillator. The small-signal ac circuit used for the noise analysis is shown in Fig. 3. Four noise sources are included in the analysis: collector-current shot noise $i_1$, base-current shot noise $i_2$, thermal-current noise in the tank resistance $i_3$ produced by the resistance $R_L = 790\,\Omega$, and thermal voltage noise in base resistance $v_4$ produced by the resistance $r_b = 7.6\,\Omega$. Circuit parasitics have not been included for clarity of comparison of analytical results and simulations.

Solving circuit equations, we derive transfer functions from a particular noise source to the output at the frequency $f_0 - \delta f$ in a manner identical to derivations presented in the Appendix. For low values of the collector bias current, the assumption of small $g_m$, i.e., $g_m \ll |sC_2|, |sC_3|$, is valid, in which case, noise transfer functions are found as follows:

$$\alpha_0^{(1)} = \frac{1}{2} \times \frac{C_3}{C_2 + C_3} \times \frac{1}{2j\omega_0 C_{\text{eff}}} \left(\frac{f_0}{\delta f}\right) \tag{3}$$

$$\alpha_0^{(2)} = \frac{1}{2} \times \frac{C_2}{C_2 + C_3} \times \frac{1}{2j\omega_0 C_{\text{eff}}} \left(\frac{f_0}{\delta f}\right) \tag{4}$$

$$\alpha_0^{(3)} = \frac{1}{2} \times \frac{1}{2j\omega_0 C_{\text{eff}}} \left(\frac{f_0}{\delta f}\right) \tag{5}$$

$$\alpha_0^{(4)} = \frac{1}{2} \times \frac{C_2 + C_3}{C_2} \times \frac{1}{2jQ} \left(\frac{f_0}{\delta f}\right) \tag{6}$$

where $\alpha_0^{(1)}, \alpha_0^{(2)}, \alpha_0^{(3)}$, and $\alpha_0^{(4)}$ refer to noise sources $i_1, i_2, i_3$, and $v_4$, respectively, $C_{\text{eff}} = C_1 + C_2C_3/(C_2 + C_3)$ is the effective tank capacitance, and $Q$ is the loaded quality factor of the tank. The phase noise contribution due to the $k$th noise source is determined by multiplying its noise spectral density by the square of the appropriate transfer function, as follows:

$$PN_0^{(k)} = N_k \left(\alpha_0^{(k)}\right)^2 \tag{7}$$

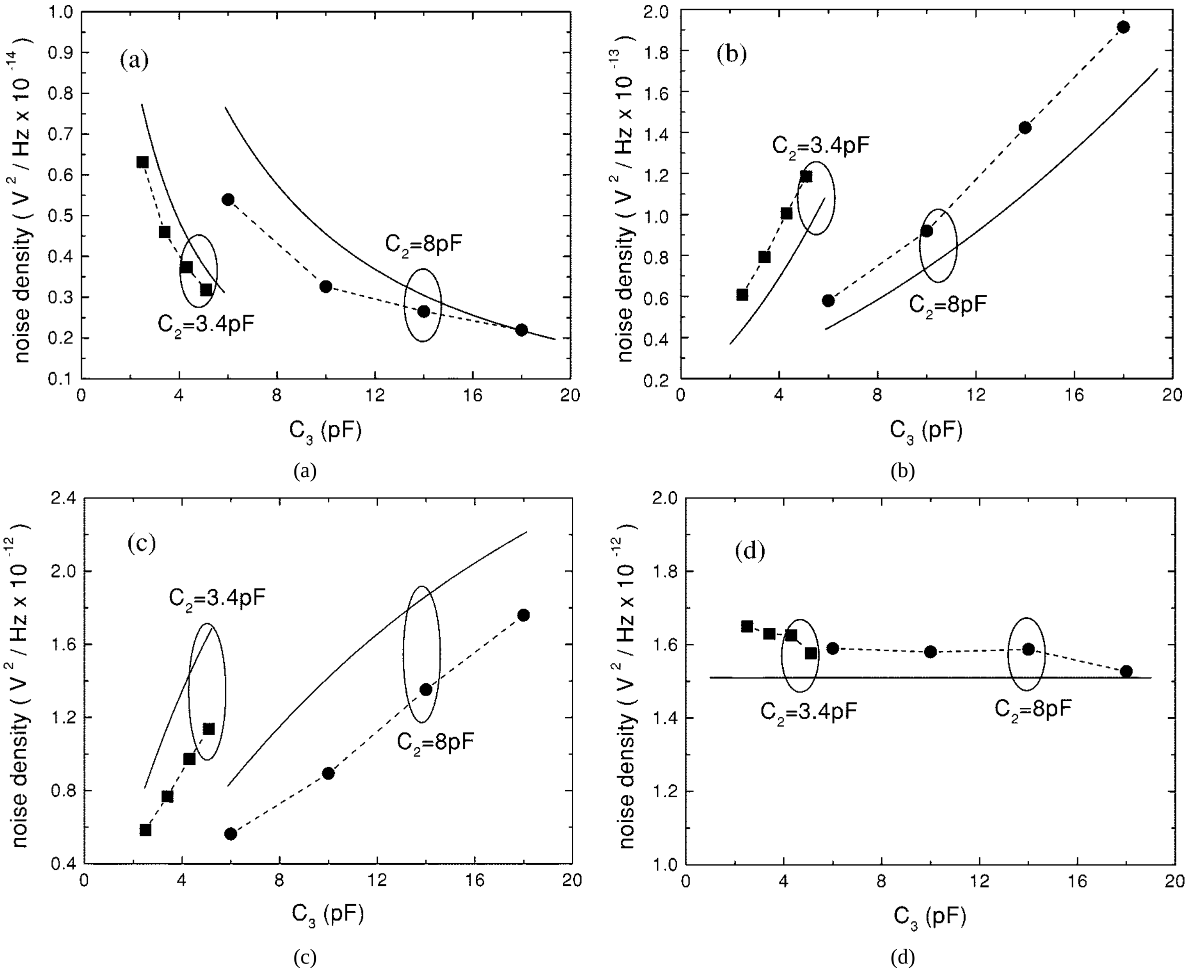


Fig. 4. Simulation results of a weakly nonlinear Colpitts oscillator for $C_2 = 3.4$ pF (squares) and $C_2 = 8$ pF (circles). Contributions due to (a) $I_B$, (b) $r_b$, (c) $I_c$, and (d) $R_L$ are shown; $f_0 = 946$ MHz and $\delta f = 25$ kHz. Ellipses group simulation and analytical curves (solid lines) for indicated values of $C_2$.

where $N_k$ refers to the spectral density of the $k$th noise source, $N_1 = 2qI_C, N_2 = 2qI_B, N_3 = 4kT/R_L,$ and $N_4 = 4kTr_b$, and the subscript 0 indicates that only the contribution from the 0th harmonic at $f_0 - \delta f$ is calculated. Equations (3)–(7) predict the dependence of phase-noise contributions on circuit parameters such as circuit capacitors, bias currents, and parasitic resistors.

To test theoretical predictions, we compare noise contributions from the 0th harmonic calculated using (3)–(7) with those simulated with the nonlinear phase-noise simulator in SpectreRF in Fig. 4. The phase-noise spectral densities at 25-kHz offset from 946-MHz carrier are plotted as a function of capacitance $C_3$ for two different values of capacitor $C_2$. Simulation results for $C_2 = 3.4$ pF and 8 pF are shown as solid squares and solid circles, respectively. Predictions of hand calculations are shown as solid lines in both cases. Results in Fig. 4 demonstrate good quantitative agreement between simulation and analytical results. The theoretical curves correctly capture trends in phase-noise variations and quantitatively lie within 30% error from simulated data.

## IV. Comparison of Simulations and Experiment

To confirm the validity of nonlinear models implemented in a simulator, we compare results of simulations and measurements on a hard-driven Colpitts oscillator. The hard-driven case represents the most challenging conditions under which one can test the performance of the simulator. A commercially available Colpitts oscillator circuit [12] was chosen for comparison purposes. Phase-noise simulations were performed with SpectreRF using measured device models and including all on-chip and off-chip parasitics. The frequency of oscillation is set at approximately 946 MHz, and the loaded $Q$ of the off-chip resonator was close to 140. The single-sideband output noise spectrum of one of the production circuits was measured using the single-sweep power spectral density measurement on the spectum analyzer. Measurement results shown in Fig. 5 indicate the total output noise power spectral density, which includes both AM- and PM-modulated components. Simulations with SpectreRF also compute the total output noise spectral density including both AM and PM components. Simulated and measured results are compared in Fig. 5 where we find a very good agreement between measurements and simulations. The flattening out of the phase noise with frequency in measured data at higher frequencies is due to noise in the buffer amplifier, which was not included in simulations.

## V. Conclusion

In conclusion, we have presented an engineering approach to the analysis of phase noise in *LC*-tuned oscillators. The basic no-

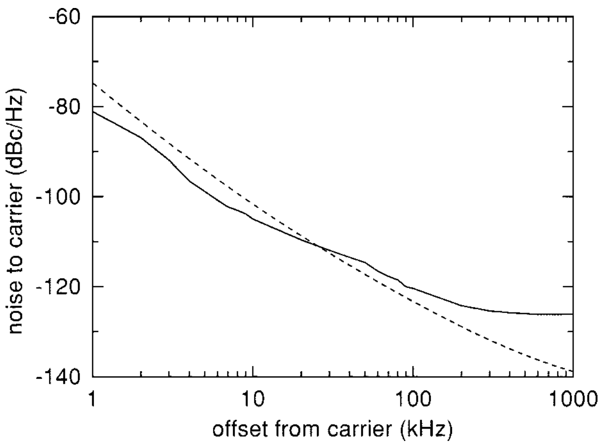


Fig. 5. Comparison of measurement (solid line) and simulation (dashed line) results for the practical Colpitts oscillator; offset is from 946-MHz carrier.

tion that has been explored in our theory is that an oscillator may be treated as a high-gain *LC* amplifier for phase-noise analysis. The techniques of oscillator noise analysis presented are intuitive and allow quantitative estimates of oscillator noise from circuit parameters. Theoretical predictions are in good agreement with nonlinear simulations using SpectreRF for weakly nonlinear oscillators. Validity of SpectreRF simulation models have been tested by comparing simulations and measurements under the most challenging conditions of a hard-driven oscillator, where we find that simulation and measurement results are in good agreement.

## APPENDIX

For an oscillator operating in the steady state at a fundamental frequency $f_0$, the nonlinear time-varying transconductance $g_m(t)$ can be represented by a sum over multiples of fundamental frequencies, as follows:

$$g_m(t) = \sum_{k=-\infty}^{+\infty} g_m^{(k)} e^{jk2\pi f_0 t} \tag{8}$$

where $k$ is the number of the harmonic. From Kirchoff's equations at the collector of the active device, we obtain

$$g_m(t)v_1(t) = nv_1(t)/Z_T \tag{9}$$

where we assumed a linear transformer coupling with a transformation ratio $1 : n$. Substituting (8) into (9) and summing current components at the fundamental frequency $f_0$, we arrive at the oscillation condition

$$g_{m,\,\mathrm{eff}} = g_m^{(0)} + g_m^{(2)} = \frac{n}{Z_T(f_0)}. \tag{10}$$

Next, we consider a noise source $i_n$ at $f_0 - \delta f$ superimposed onto the oscillating waveform as shown in Fig. 1. The nonlinear transconductance $g_m(t)$ modulates this noise source to create noise sidebands at $f_0 - \delta f$ and $-f_0 - \delta f$ in the output waveform. At the input to the active device, we denote rms voltages of the former sideband by $v_1^{(+)}$ and of the latter by $v_1^{(-)}$. Similarly to (9), we write Kirchoff's equations in the presence of the noise generator $i_n$ for $f_0 - \delta f$ and $-f_0 - \delta f$ as

$$g_m^{(0)} v_1^{(+)} + g_m^{(2)} v_1^{(-)} + i_n = \frac{nv_1^{(+)}}{Z_T(f_0)} \tag{11}$$

and

$$g_m^{(0)} v_1^{(-)} + g_m^{(2)} v_1^{(+)} = \frac{nv_1^{(-)}}{Z_T(f_0)} \tag{12}$$

respectively. Solving for $v_1^{(+)} + v_1^{(-)}$ and $v_1^{(+)} - v_1^{(-)}$, we find

$$v_1^{(+)} + v_1^{(-)} = \frac{-i_n}{g_m^{(0)} + g_m^{(2)} - n/Z_T(f_0)} \tag{13}$$

and

$$v_1^{(+)} - v_1^{(-)} = \frac{-i_n}{g_m^{(0)} - g_m^{(2)} - n/Z_T(f_0)}. \tag{14}$$

Using the oscillation condition, (10), in (13) and (14), we find that in steady state the right-hand side of (13) peaks very sharply, and is much greater than the right hand side of (14). This leads to $v_1^{(+)} = v_1^{(-)}$, and to the expression for a single-sided transfer function, from $i_n$ to $v_1^{(+)}$, given by

$$v_1^{(+)} = \frac{1}{2} \times \frac{-i_n}{g_{m,\mathrm{eff}}^{(0)} - n/Z_T(f_0)}. \tag{15}$$

To obtain the output transfer function $\alpha_0$, we use $v_0^{(+)} = nv_1^{(+)}$ to arrive at

$$\alpha_0 = \frac{1}{2} \times \frac{R_L}{2jQ} \left(\frac{f_0}{\delta f}\right). \tag{16}$$

This is the noise transfer function from the noise tone at $f_0 - \delta f$ to the output noise of the oscillator at the same frequency. This transfer function explicitly contains a factor of 1/2, which is due to noise translation by $-2f_0$ to the negative frequencies at the output.

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