EECS 242

Two-Port Power Gain

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Two-Port Power and Scattering Parameters

- The power flowing into a two-port can be represented by

$$P_{in} = \frac{|V_1^+|^2}{2Z_0}(1 - |\Gamma_{in}|^2)$$

- The power flowing to the load is likewise given by

$$P_L = \frac{|V_2^-|^2}{2Z_0}(1 - |\Gamma_L|^2)$$

- We can solve for V_1^+ using circuit theory

$$V_1^+ + V_1^- = V_1^+(1 + \Gamma_{in}) = \frac{Z_{in}}{Z_{in} + Z_S} V_S$$

- In terms of the input and source reflection coefficient

$$Z_{in} = \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} Z_0 \qquad Z_S = \frac{1 + \Gamma_S}{1 - \Gamma_S} Z_0$$

Two-Port Incident Wave

- Solve for V_1^+

$$V_1^+(1+\Gamma_{in}) = \frac{V_S(1+\Gamma_{in})(1-\Gamma_S)}{(1+\Gamma_{in})(1-\Gamma_S)+(1+\Gamma_S)(1-\Gamma_{in})}$$

$$V_1^+ = \frac{V_S}{2}\frac{1-\Gamma_S}{1-\Gamma_{in}\Gamma_S}$$

- The voltage incident on the load is given by

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ + S_{22}\Gamma_L V_2^-$$

$$V_2^- = \frac{S_{21}V_1^+}{1-S_{22}\Gamma_L}$$

$$P_L = \frac{|S_{21}|^2\left|V_1^+\right|^2}{|1-S_{22}\Gamma_L|^2}\frac{1-|\Gamma_L|^2}{2Z_0}$$

Operating Gain and Available Power

- The operating power gain can be written in terms of the two-port s-parameters and the load reflection coefficient

$$G_p = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2 (1 - |\Gamma_{in}|^2)}$$

- The available power can be similarly derived from V_1^+

$$P_{avs} = P_{in}|_{\Gamma_{in}=\Gamma_S^*} = \frac{\left|V_{1a}^+\right|^2}{2Z_0}(1 - |\Gamma_S^*|^2)$$

$$V_{1a}^+ = V_1^+\Big|_{\Gamma_{in}=\Gamma_S^*} = \frac{V_S}{2}\frac{1 - \Gamma_S^*}{1 - |\Gamma_S|^2}$$

$$P_{avs} = \frac{|V_S|^2}{8Z_0}\frac{|1 - \Gamma_S|^2}{1 - |\Gamma_S|^2}$$

Transducer Gain

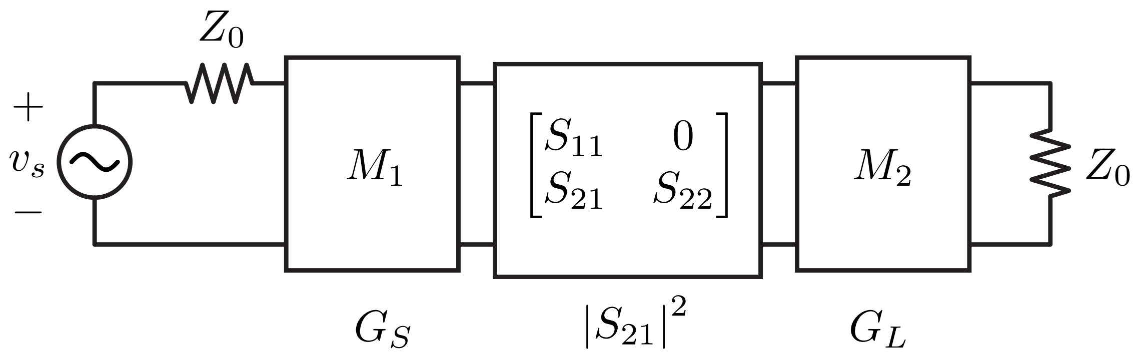
- The transducer gain can be easily derived

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)(1 - |\Gamma_S|^2)}{|1 - \Gamma_{in}\Gamma_S|^2 \, |1 - S_{22}\Gamma_L|^2}$$

- Note that as expected, G_T is a function of the two-port s-parameters and the load and source impedance.
- If the two port is connected to a source and load with impedance Z_0, then we have $\Gamma_L = \Gamma_S = 0$ and

$$G_T = |S_{21}|^2$$

Unilateral Gain



- If $S_{12} \approx 0$, we can simplify the expression by just assuming $S_{12} = 0$. This is the *unilateral* assumption

$$G_{TU} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = G_S \, |S_{21}|^2 \, G_L$$

- The gain partitions into three terms, which can be interpreted as the gain from the source matching network, the gain of the two port, and the gain of the load.
- In reality the source/load matching network are passive and hopefully lossless, so the power gain is 1 or less, but by virtue of the matching network we can change the gain of the two-port.

Maximum Unilateral Gain

- We know that the maximum gain occurs for the biconjugate match

$$\Gamma_S = S_{11}^*$$

$$\Gamma_L = S_{22}^*$$

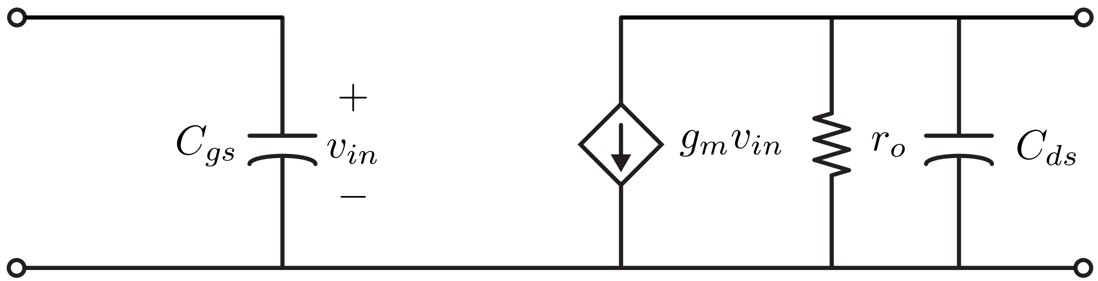
$$G_{S,max} = \frac{1}{1 - |S_{11}|^2}$$

$$G_{L,max} = \frac{1}{1 - |S_{22}|^2}$$

$$G_{TU,max} = \frac{|S_{21}|^2}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)}$$

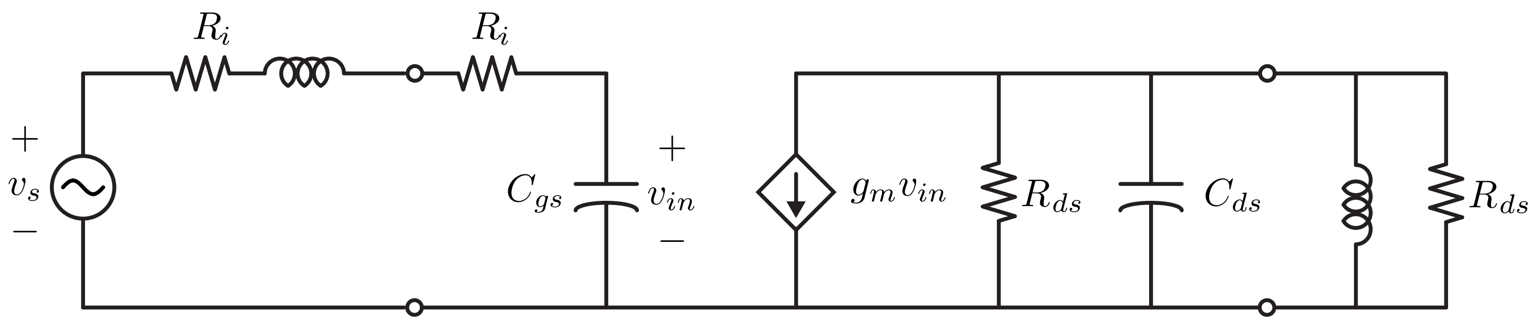
- Note that if $|S_{11}| = 1$ of $|S_{22}| = 1$, the maximum gain is infinity. This is the unstable case since $|S_{ii}| > 1$ is potentially unstable.

Ideal MOSFET



- The AC equivalent circuit for a MOSFET at low to moderate frequencies is shown above. Since $|S_{11}| = 1$, this circuit has infinite power gain. This is a trivial fact since the gate capacitance cannot dissipate power whereas the output can deliver real power to the load.

Real MOSFET



- A more realistic equivalent circuit is shown above. If we make the unilateral assumption, then the input and output power can be easily calculated. Assume we conjugate match the input/output

$$P_{avs} = \frac{|V_S|^2}{8R_i}$$

$$P_L = \Re(\tfrac{1}{2} I_L V_L^*) = \tfrac{1}{2} \left| \frac{g_m V_1}{2} \right|^2 R_{ds}$$

$$G_{TU,max} = g_m^2 R_{ds} R_i \left| \frac{V_1}{V_S} \right|^2$$

Real MOSFET (cont)

- At the center resonant frequency, the voltage at the input of the FET is given by

$$V_1 = \frac{1}{j\omega C_{gs}} \frac{V_S}{2R_i}$$

$$G_{TU,max} = \frac{R_{ds}}{R_i} \frac{(g_m/C_{gs})^2}{4\omega^2}$$

- This can be written in terms of the device unity gain frequency f_T

$$G_{TU,max} = \frac{1}{4} \frac{R_{ds}}{R_i} \left(\frac{f_T}{f} \right)^2$$

- The above expression is very insightful. To maximum power gain we should maximize the device f_T and minimize the input resistance while maximizing the output resistance.

Design for Gain

- So far we have only discussed power gain using bi-conjugate matching. This is possible when the device is unconditionally stable. In many case, though, we'd like to design with a potentially unstable device.
- Moreover, we would like to introduce more flexibility in the design. We can trade off gain for
  - bandwidth
  - noise
  - gain flatness
  - linearity
  - etc.
- We can make this tradeoff by identifying a range of source/load impedances that can realize a given value of power gain. While maximum gain is acheived for a single point on the Smith Chart, we will find that a lot more flexibility if we back-off from the peak gain.

Unilateral Design

- No real transistor is unilateral. But most are predominantly unilateral, or else we use cascades of devices (such as the cascode) to realize such a device.
- The *unilateral figure of merit* can be used to test the validity of the unilateral assumption

$$U_m = \frac{|S_{12}|^2\,|S_{21}|^2\,|S_{11}|^2\,|S_{22}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

- It can be shown that the transducer gain satisfies the following inequality

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$

- Where the actual power gain G_T is compared to the power gain under the unilateral assumption G_{TU}. If the inequality is tight, say on the order of $0.1\,\mathrm{dB}$, then the amplifier can be assumed to be unilateral with negligible error.

Gain Circles

- We now can plot gain circles for the source and load. Let

$$g_S = \frac{G_S}{G_{S,max}}$$

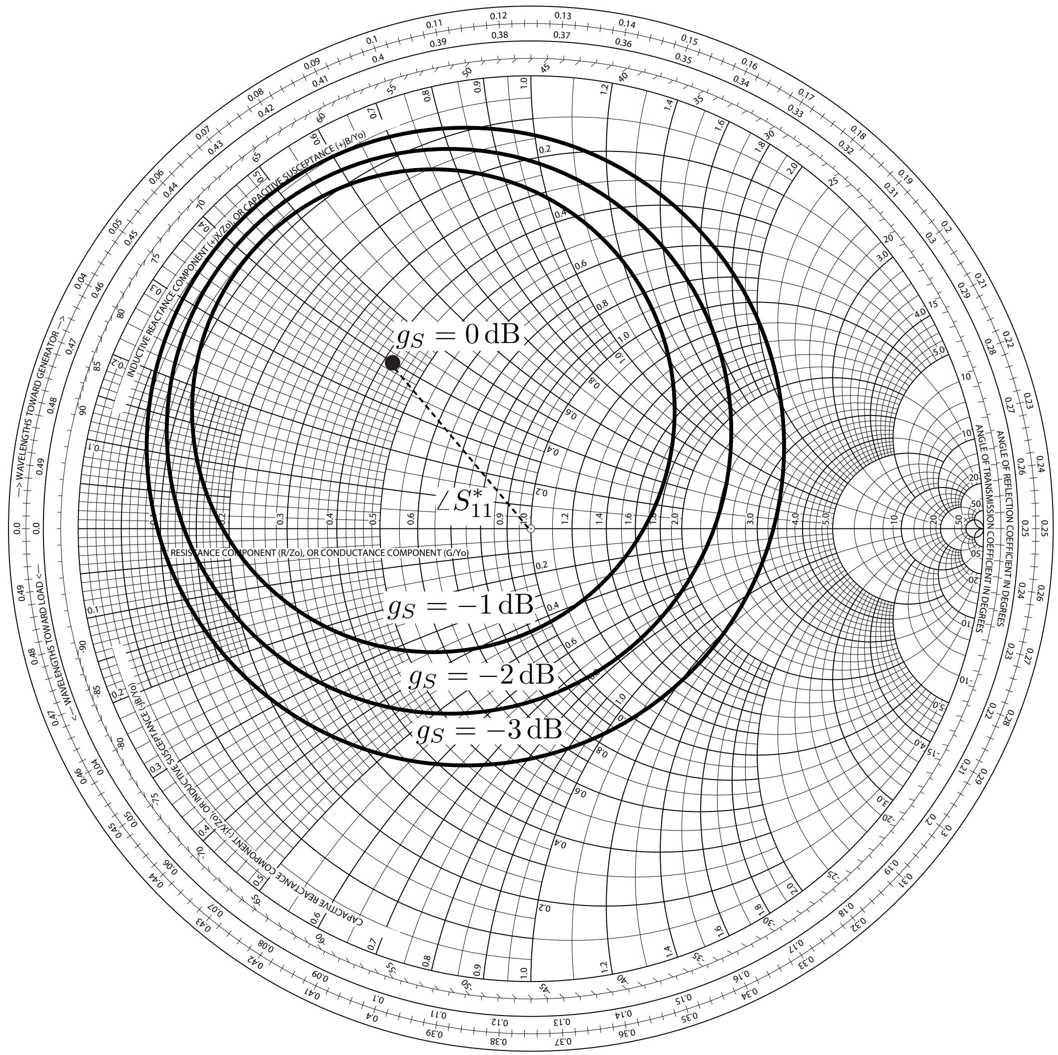
$$g_L = \frac{G_L}{G_{L,max}}$$

- By definition, $0 \le g_S \le 1$ and $0 \le g_L \le 1$. One can show that a fixed value of g_S represents a circle on the Γ_S plane

$$\left| \Gamma_S - \frac{S_{11}^* g_S}{|S_{11}|^2 (g_S - 1) + 1} \right| = \left| \frac{\sqrt{1 - g_S}(1 - |S_{11}|^2)}{|S_{11}|^2 (g_S - 1) + 1} \right|$$

- More simply, $|\Gamma_S - C_S| = R_S$. A similar equation can be derived for the load. Note that for $g_S = 1$, $R_S = 0$, and $C_S = S_{11}^*$ corresponding to the maximum gain.

Gain Circles (cont)



- All gain circles lie on the line given by the angle of S_{ii}^*. We can select any desired value of source/load reflection coefficient to acheive the desired gain. To minimize the impedance mismatch, and thus maximize the bandwidth, we should select a point closest to the origin.

Extended Smith Chart

- For $|\Gamma| > 1$, we can still employ the Smith Chart if we make the following mapping. The reflection coefficient for a negative resistance is given by

$$\Gamma(-R + jX) = \frac{-R + jX - Z_0}{-R + jX + Z_0} = \frac{(R + Z_0) - jX}{(R - Z_0) - jX}$$

$$\frac{1}{\Gamma^*} = \frac{(R - Z_0) + jX}{(R + Z_0) + jX}$$

- We see that Γ can be mapped to the unit circle by taking $1/\Gamma^*$ and reading the resistance value (and noting that it's actually negative).

Potentially Unstable Unilateral Amplifier

- For a unilateral two-port with $|S_{11}| > 1$, we note that the input impedance has a negative real part. Thus we can still design a stable amplifier as long as the source resistance is larger than $\Re(Z_{in})$

$$\Re(Z_S) > |\Re(Z_{in})|$$

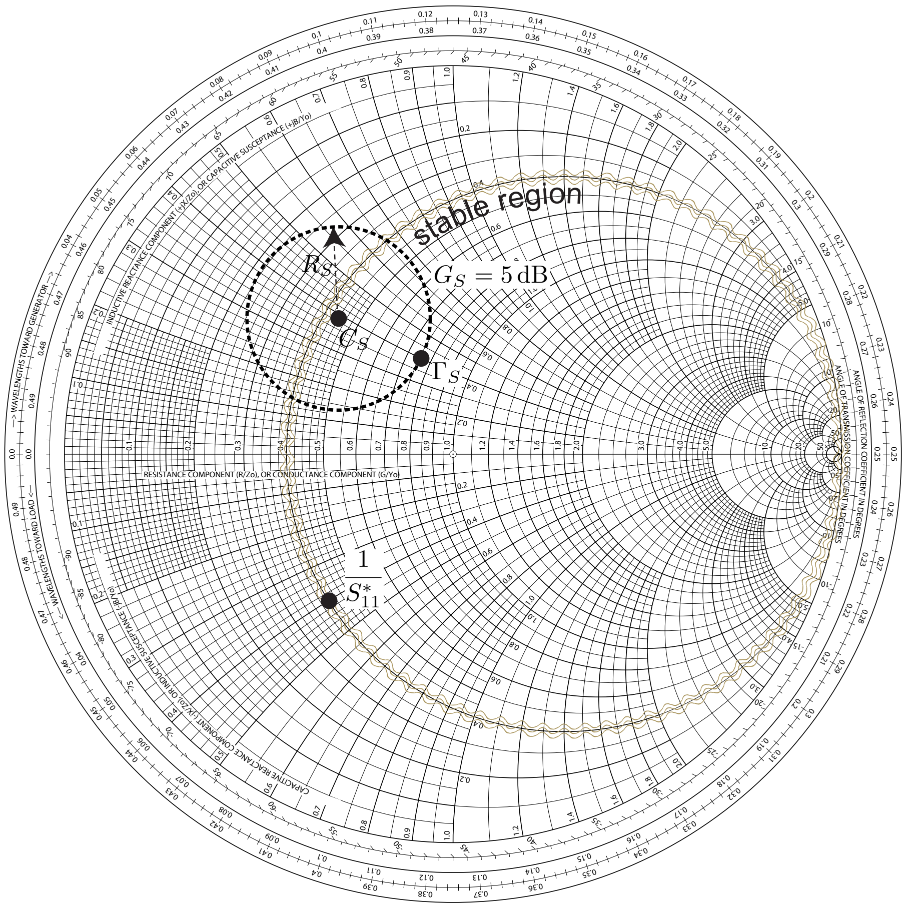
- The same is true of the load impedance if $|S_{22}| > 1$. Thus the design procedure is identical to before as long as we avoid source or load reflection coefficients with real part less than the critical value.

Pot. Unstable Unilateral Amp Example

- Consider a transistor with the following S-Parameters

$$S_{11} = 2.02\angle - 130.4^\circ \qquad S_{12} = 0$$

$$S_{22} = 0.50\angle - 70^\circ \qquad S_{21} = 5.00\angle 60^\circ$$



- Since $|S_{11}| > 1$, the amplifier is potentially unstable. We begin by plotting $1/S_{11}^*$ to find the negative real input resistance.
- Now any source inside this circle is stable, since $\Re(Z_S) > \Re(Z_{in})$.
- We also draw the source gain circle for $G_S = 5\,\mathrm{dB}$.

Amp Example (cont)

- The input impedance is read off the Smith Chart from $1/S_{11}^*$. Note the real part is interpreted as negative

$$Z_{in} = 50(-0.4 - 0.4j)$$

- The $G_S = 5\,\mathrm{dB}$ gain circle is calculated as follows

$$g_S = 3.15(1 - |S_{11}|^2)$$

$$R_S = \frac{\sqrt{1 - g_S}(1 - |S_{11}|^2)}{1 - |S_{11}|^2(1 - g_S)} = 0.236$$

$$C_S = \frac{g_S S_{11}^*}{1 - |S_{11}|^2(1 - g_S)} = -.3 + 0.35j$$

- We can select any point on this circle and obtain a stable gain of $5\,\mathrm{dB}$. In particular, we can pick a point near the origin (to maximize the BW) but with as large of a real impedance as possible:

$$Z_S = 50(0.75 + 0.4j)$$

Bilateral Amp Design

- In the bilateral case, we will work with the power gain G_p. The transducer gain is not used since the source impedance is a function of the load impedaance. G_p, on the other hand, is only a function of the load.

$$G_p = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{\left(1 - \left|\frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}\right|^2\right) |1 - S_{22}\Gamma_L|^2} = |S_{21}|^2 g_p$$

- It can be shown that g_p is a circle on the Γ_L plane. The radius and center are given by

$$R_L = \frac{\sqrt{1 - 2K|S_{12}S_{21}|g_p + |S_{12}S_{21}|^2 g_p^2}}{\left|-1 - |S_{22}|^2 g_p + |\Delta|^2 g_p\right|^2}$$

$$C_L = \frac{g_p(S_{22}^* - \Delta^* S_{11})}{1 + g_p(|S_{22}|^2 - |\Delta|^2)}$$

Bilateral Amp (cont)

- We can also use this formula to find the maximum gain. We know that this occurs when $R_L = 0$, or

$$1 - 2K|S_{12}S_{21}|g_{p,max} + |S_{12}S_{21}|^2 g_{p,max}^2 = 0$$

$$g_{p,max} = \frac{1}{|S_{12}S_{21}|}\left(K - \sqrt{K^2 - 1}\right)$$

$$G_{p,max} = \left|\frac{S_{21}}{S_{12}}\right|\left(K - \sqrt{K^2 - 1}\right)$$

- The design procedure is as follows
  1. Specify g_p
  2. Draw operating gain circle.
  3. Draw load stability circle. Select Γ_L that is in the stable region and not too close to the stability circle.
  4. Draw source stability circle.
  5. To maximize gain, calculate Γ_{in} and check to see if $\Gamma_S = \Gamma_{in}^*$ is in the stable region. If not, iterate on Γ_L or compromise.