Injection Locking

EECS 242 Lecture 26

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Outline

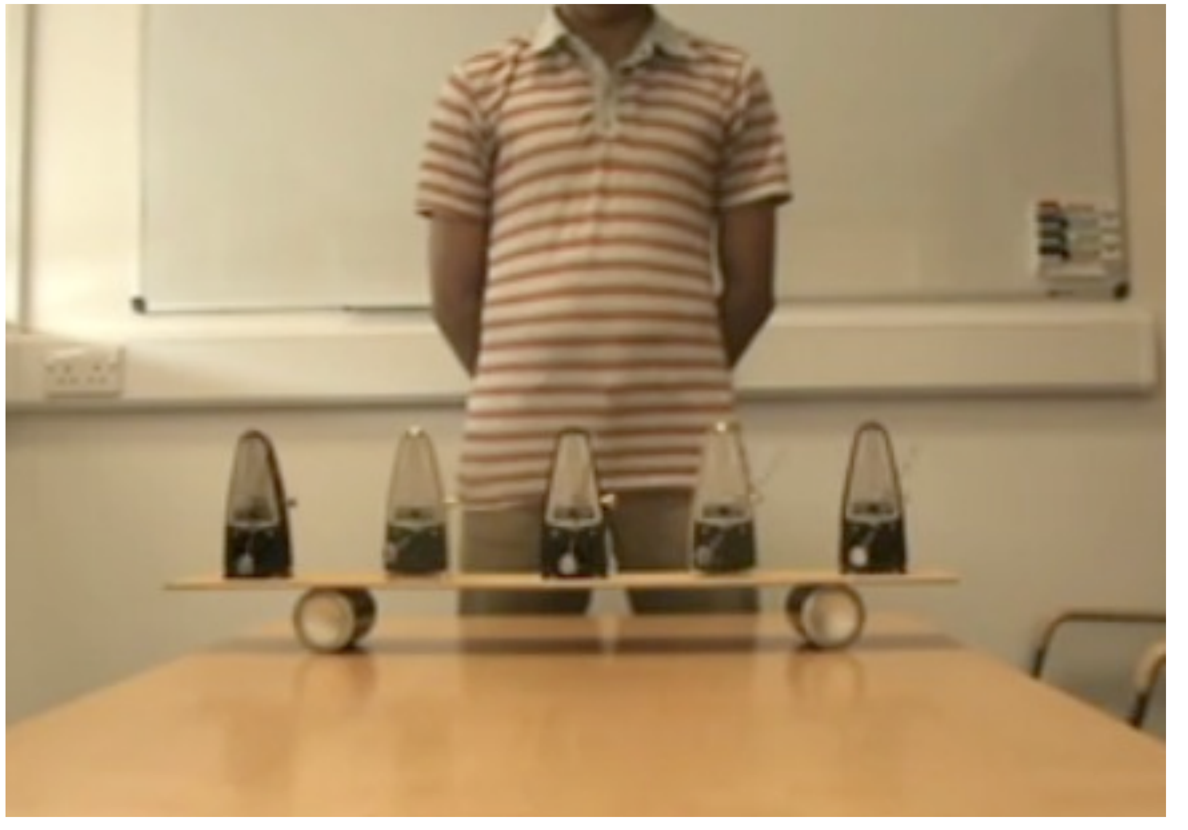
- Injection Locking
  - Adler's Equation (locking range)
  - Extension to large signals
- Examples:
  - GSM CMOS PA
  - Low Power Transmitter
  - Dual Mode Oscillators
  - Clock distribution
- Quadrature Locked Oscillators
- Injection locked dividers

Injection Locking

http://www.youtube.com/watch?v=IBgq-_NJCl0

- Injection locking is also known as frequency entrainment or synchronization
- Many natural examples including
  - pendulum clocks on the same wall observed to synchronize over time
  - fireflies put on a good light show
- Injection locking can be deliberate or unwanted

Injection Locking Video Demonstration

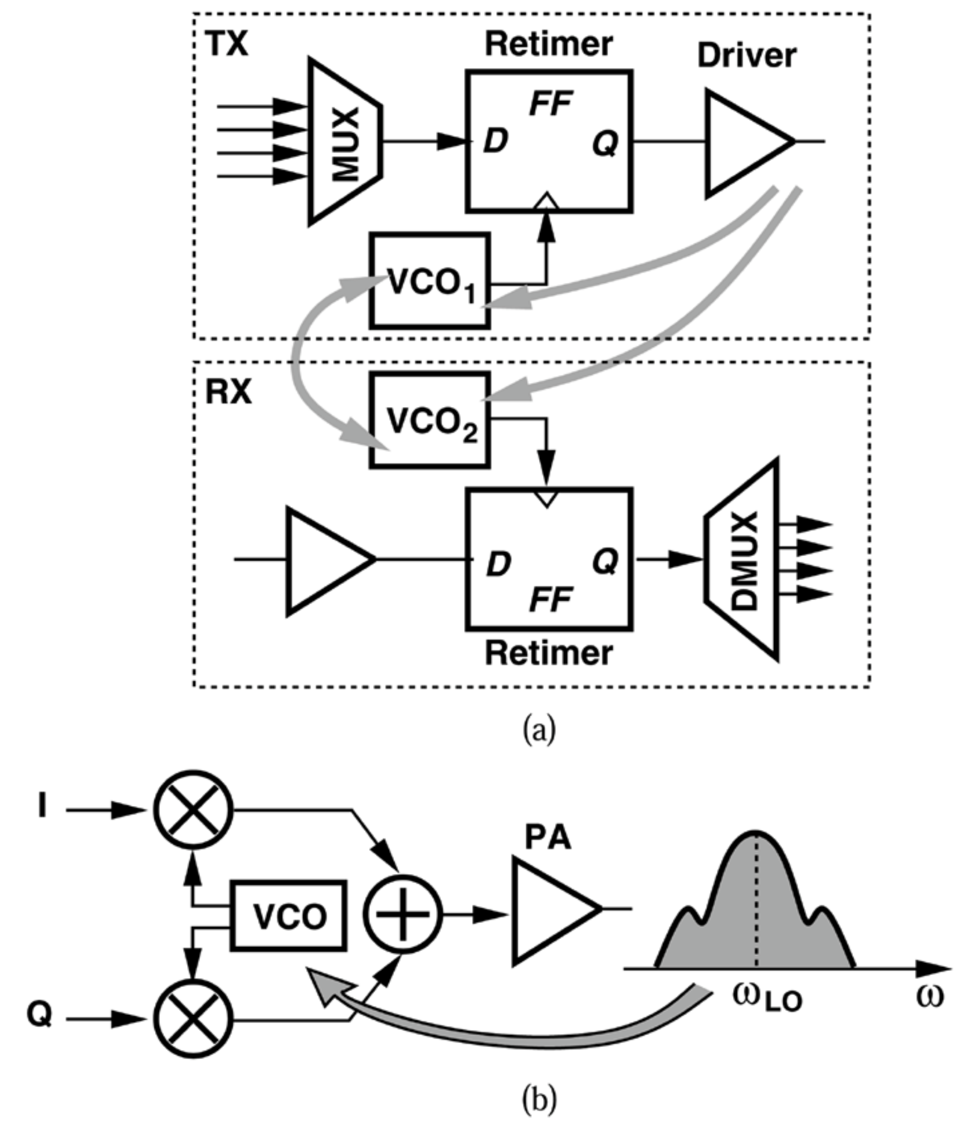


http://www.youtube.com/watch?v=W1TMZASCR-I

- Several metronomes (similar to pendulums) are initially excited in random phases. The oscillation frequencies are presumably very close but vary slightly due to manufacturing imperfections
- When placed on a rigid surface, the metronomes oscillate independently.
- When placed on flexible table with "springs" (coke cans), they couple to one another and injection lock.

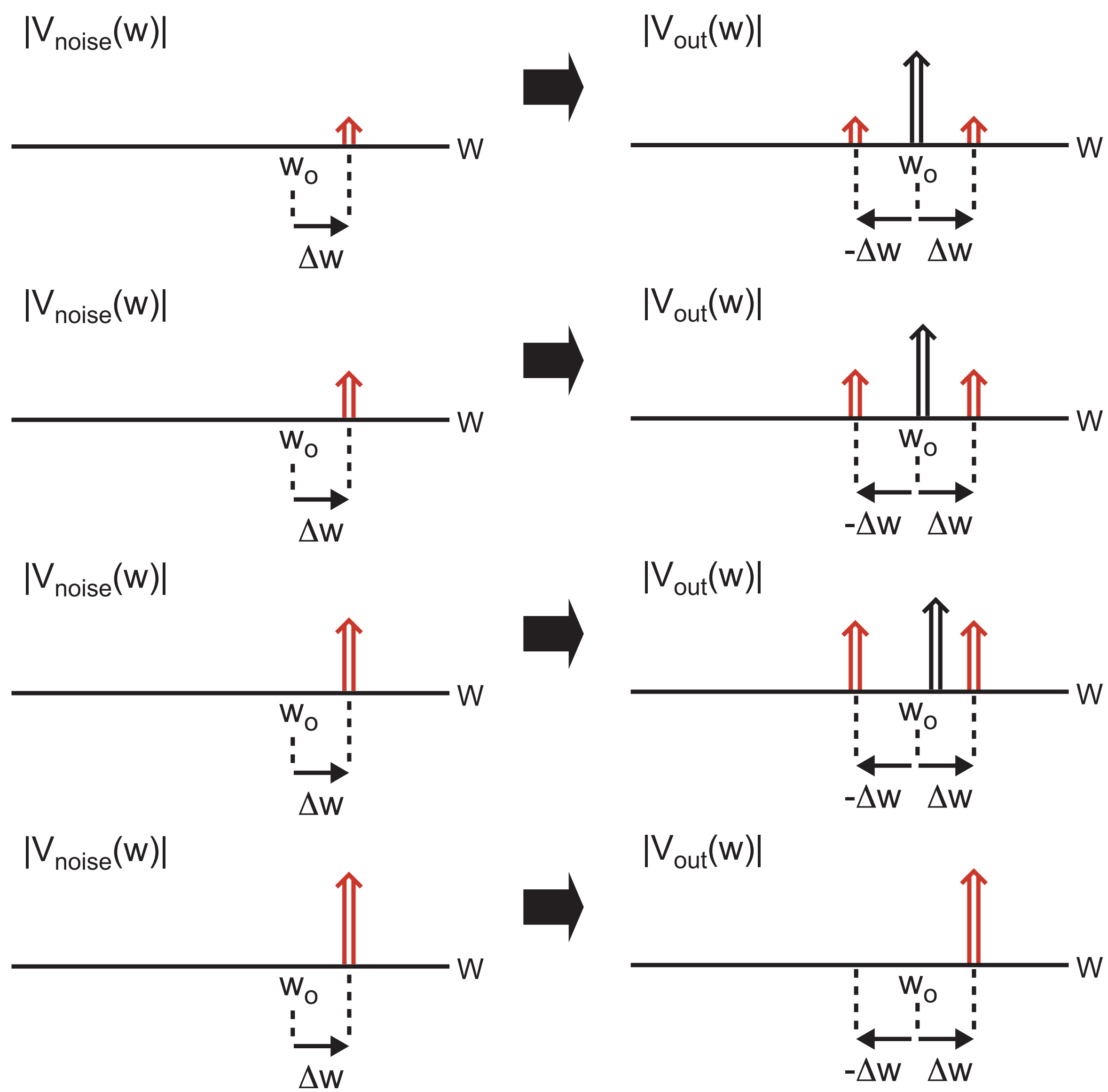
Unwanted Injection Pulling/Locking

- One of the difficulties in designing a fully integrated transceiver is exactly due to pulling / pushing
- If the injection signal is strong enough, it will lock the source. Otherwise it will "pull" the source and produce unwanted modulation
- In the first example, the transmitter is locked to a XTAL whereas the receiver is locked to the data clock. Unwanted coupling (package, substrate, Vdd/Gnd) can cause pulling.
- A PA is a classic source of trouble in a direct-conversion transmitter



Source: [Razavi]

Injection Locking is Non-Linear



Source: M. Perrott
MIT OCW 6.976

- For weak injection, you get a response at both side-bands
- As the injection is increased, it begins to “pull” the oscillator
- Eventually, for large enough injection, the oscillation locks to the injection signal

Injection Locking in LC Tanks

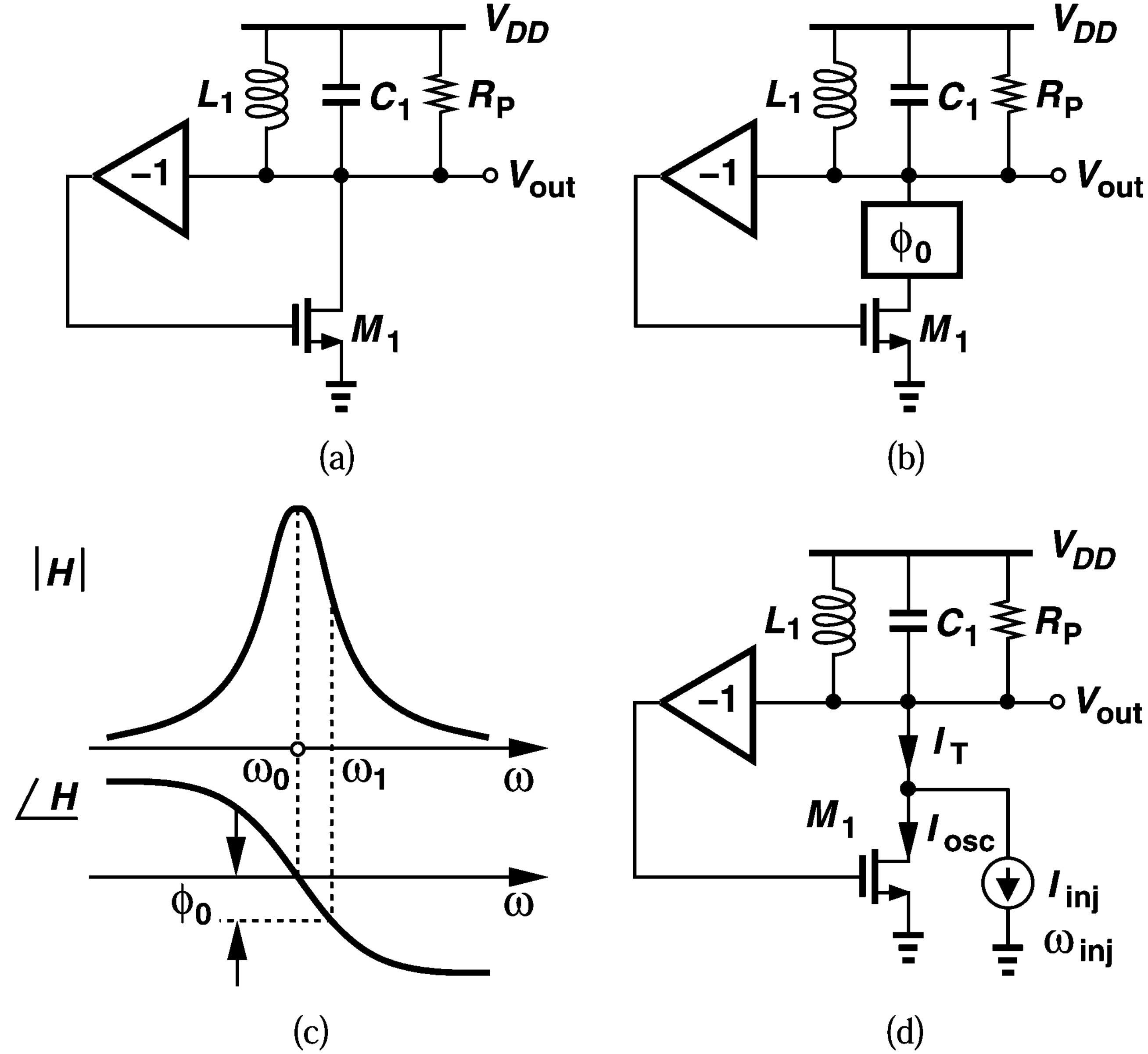
- Consider (a) a free-running oscillator consisting of an ideal positive feedback amplifier and an LC tank.
- Now suppose (b) we insert a phase shift in the loop. We know this will cause the oscillation frequency to (c) shift since the loop gain has to have exactly 2π phase shift (or multiples)

$$G_m Z_T(\omega_0) = g_m R = 1$$

$$G_m e^{j\phi_0} Z_T(\omega_1) = 1$$

$$G_m e^{j\phi_0} |Z_T(\omega_1)| e^{-j\phi_0} = 1$$

$$\angle Z_T(\omega_1) = -\phi_0$$



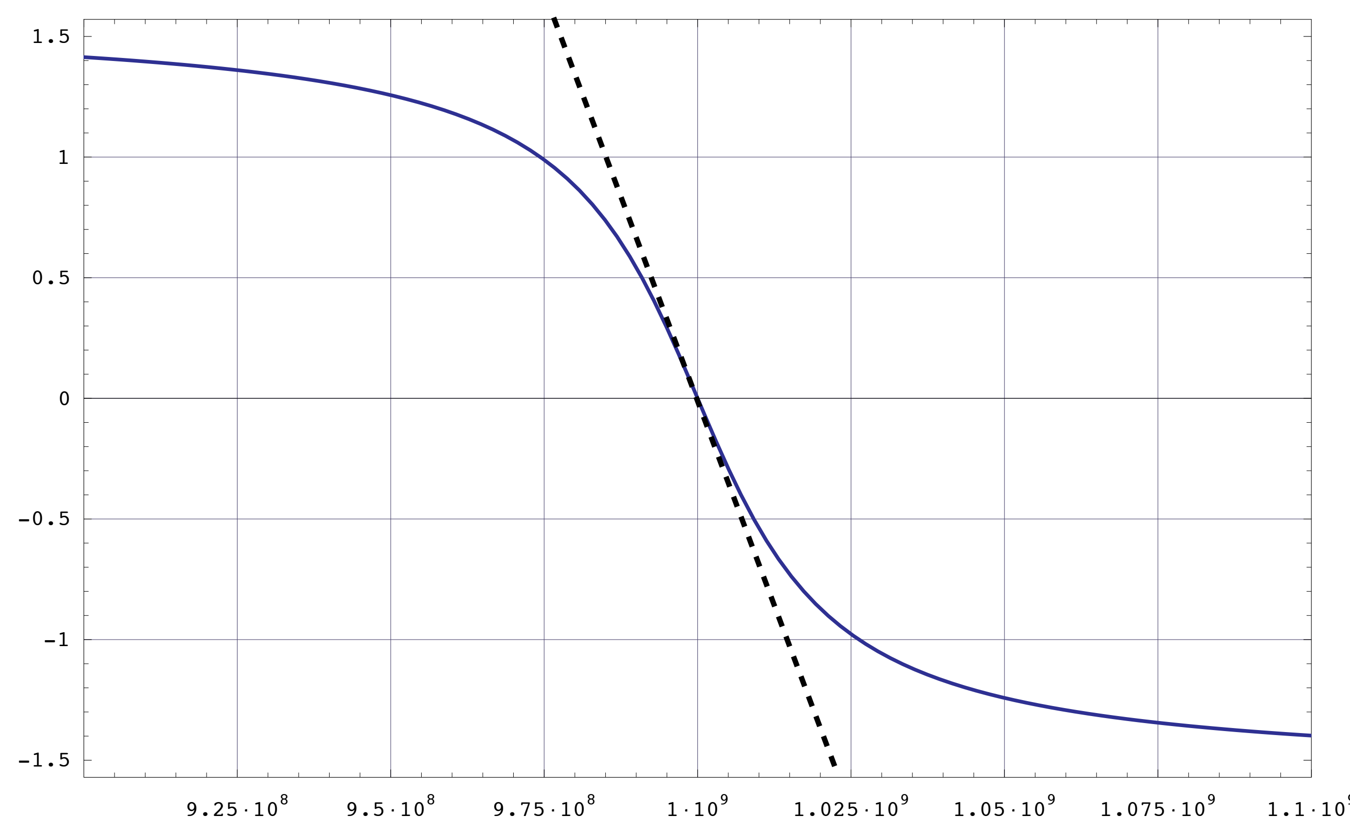
Source: [Razavi]

Injection Locking in LC Tanks [cont]

- A phase shift in the tank will cause the oscillation frequency to change in order to compensate for the phase shift through the tank impedance.
- The oscillation frequency is no longer at the resonant frequency of the tank. Note that the oscillation amplitude must also change since the loop gain is now different (tank impedance is lower)
- Maximum phase shift that the tank can provide is ± 90°
- In a high Q tank, the frequency shift is relatively small since

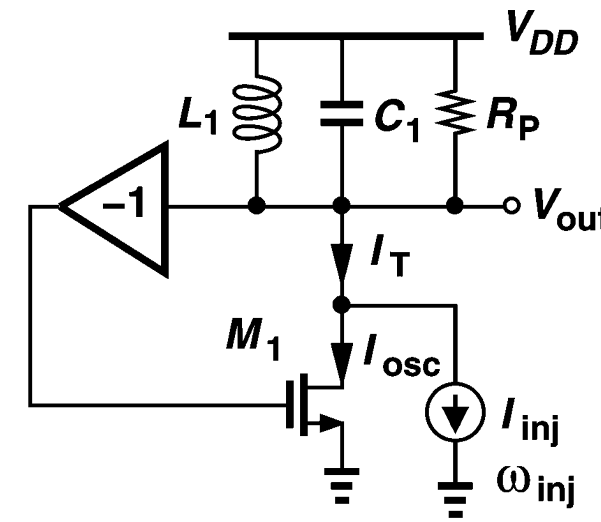
$$Q = \frac{\omega_0}{2}\frac{d\phi}{d\omega}$$

$$\Delta\omega \approx \frac{\omega_0}{2}\frac{\Delta\phi}{Q}$$



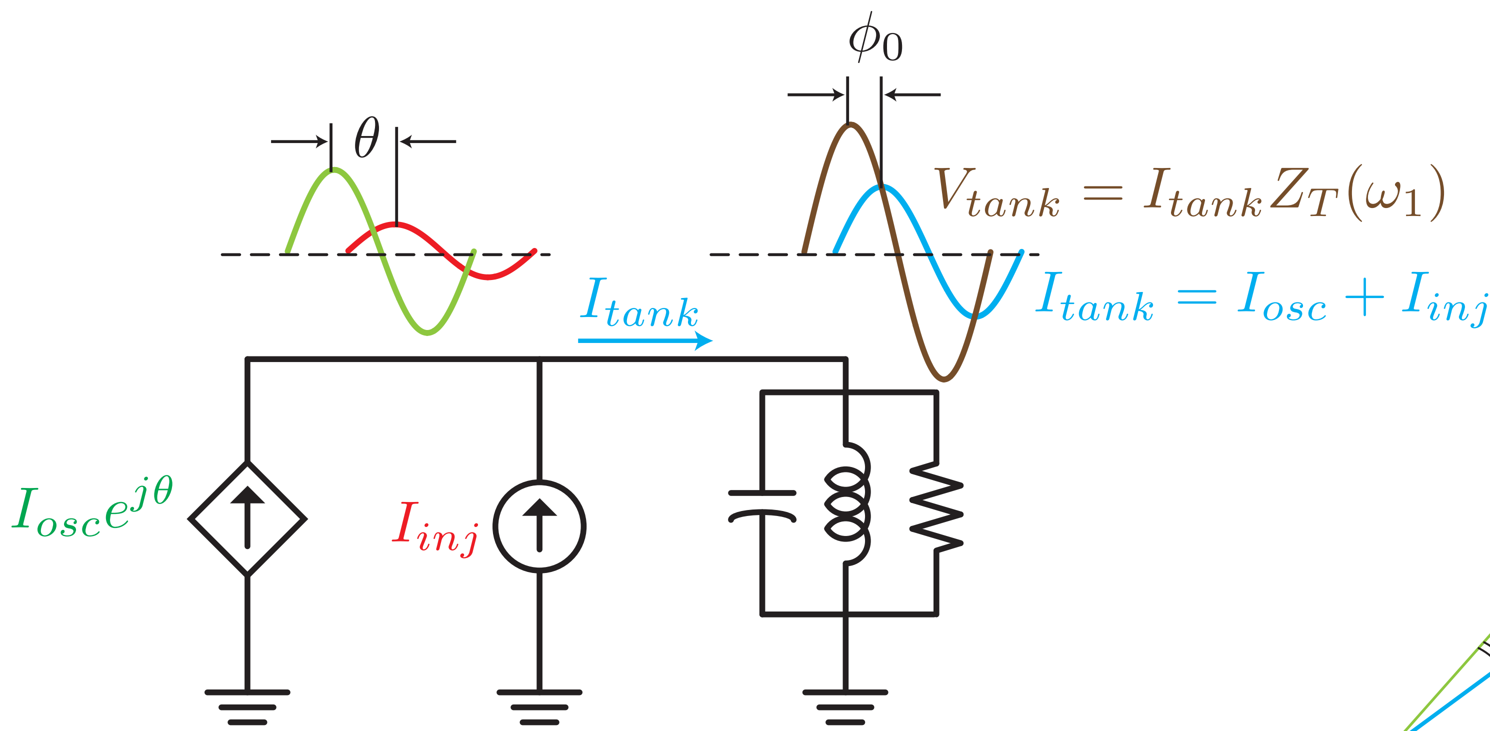
Phase Shift for Injected Signal

- It's interesting to observe that if a signal is injected into the circuit, then the tank current is a sum of the injected and transistor current.
- Assume the oscillator "locks" onto the injected current and oscillates at the same frequency.
- Since the locking signal is not in general at the resonant center frequency, the tank introduces a phase shift
- In order for the oscillator loop gain to be equal to unity with zero phase shift, the sum of the current of the transistor and the injected currents must have the proper phase shift to compensate for the tank phase shift.
- We see that the oscillator current, tank current, and injected current all have different phases

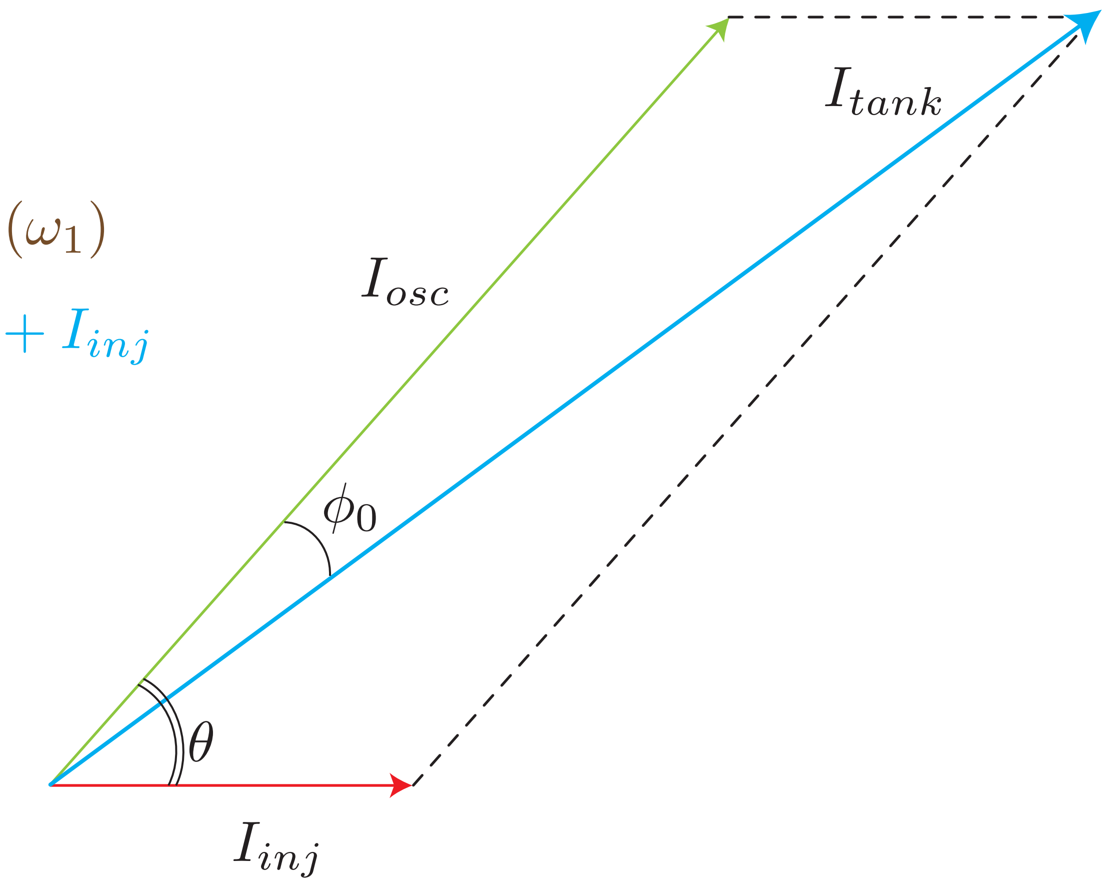


Source: [Razavi]

Injection Locked Oscillator Phasors



$$\angle I_{osc} = \angle V_{tank} = \angle I_{tank} + \phi_0$$



- Note that the frequency of the injection signal determines the extra phase shift Φ_0 of the tank. This is fixed by the frequency offset.
- The current from the transistor is fed by the tank voltage, which by definition the tank current times the tank impedance, which introduces Φ_0 between the tank current/voltage.
- The angle between the injected current and the oscillator current θ must be such that their sum aligns with the tank current.

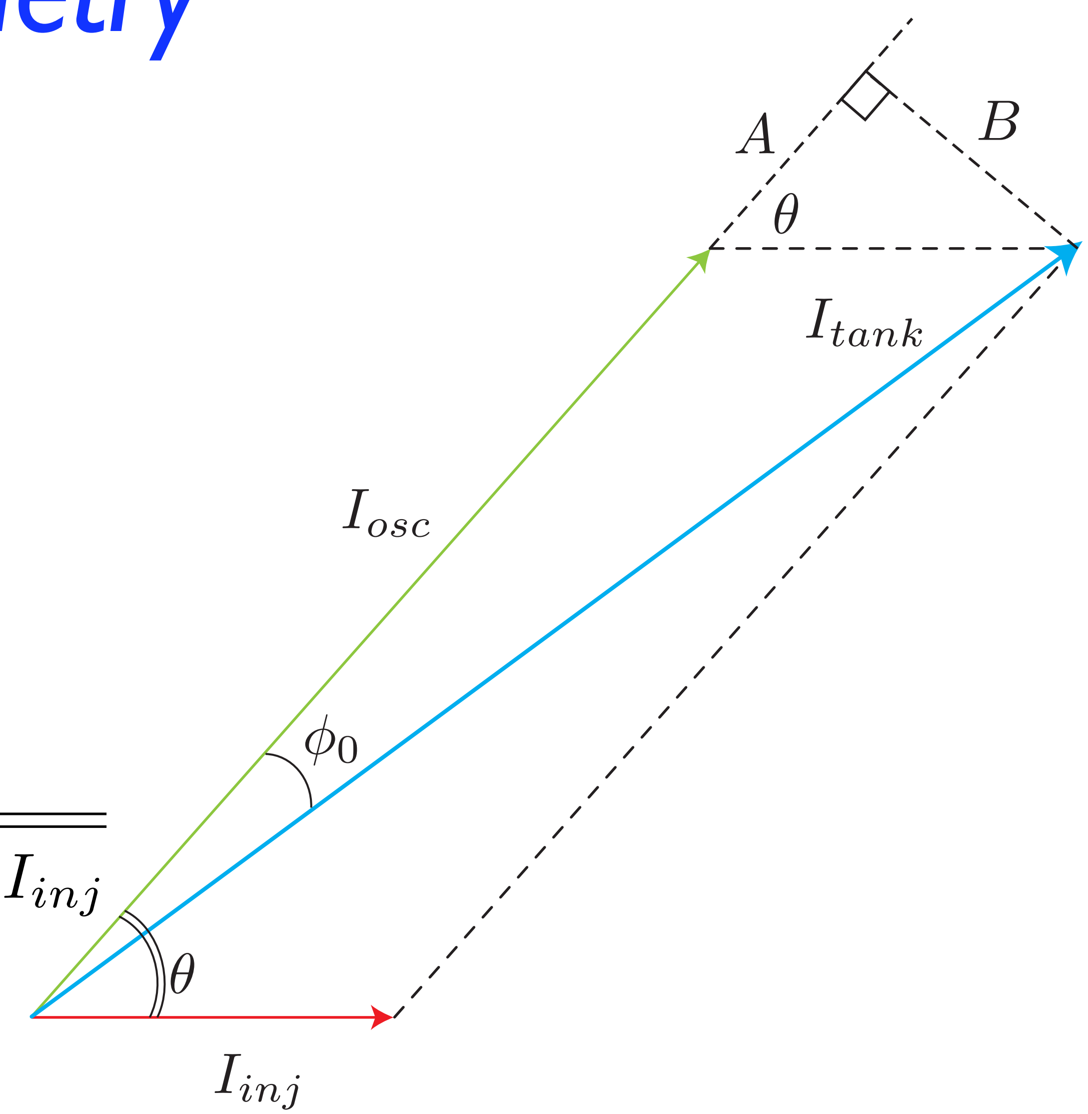
Injection Geometry

$$\sin\phi_0 = \frac{B}{I_{tank}}$$

$$\cos(\pi/2-\theta) = \sin(\theta) = \frac{B}{I_{inj}}$$

$$\sin\phi_0 = \frac{I_{inj}}{I_{tank}}\sin(\theta)$$

$$\sin\phi_0 = \frac{I_{inj}\sin(\theta)}{|I_{osc}e^{j\theta}+I_{inj}|} = \frac{I_{inj}\sin(\theta)}{\sqrt{I_{osc}^2+I_{inj}^2+2\cos\theta I_{osc}I_{inj}}}$$



- The geometry of the problem implies the following constraints on the injected current amplitude relative to the oscillation amplitude.
- The maximum value of the rhs occurs at:

$$\cos\theta = \frac{-I_{inj}}{I_{osc}} \qquad \sin\theta = \frac{\sqrt{I_{osc}^2 - I_{inj}^2}}{I_{osc}} \qquad \sin\phi_{0,max} = \frac{I_{inj}}{I_{osc}}$$

Locking Range

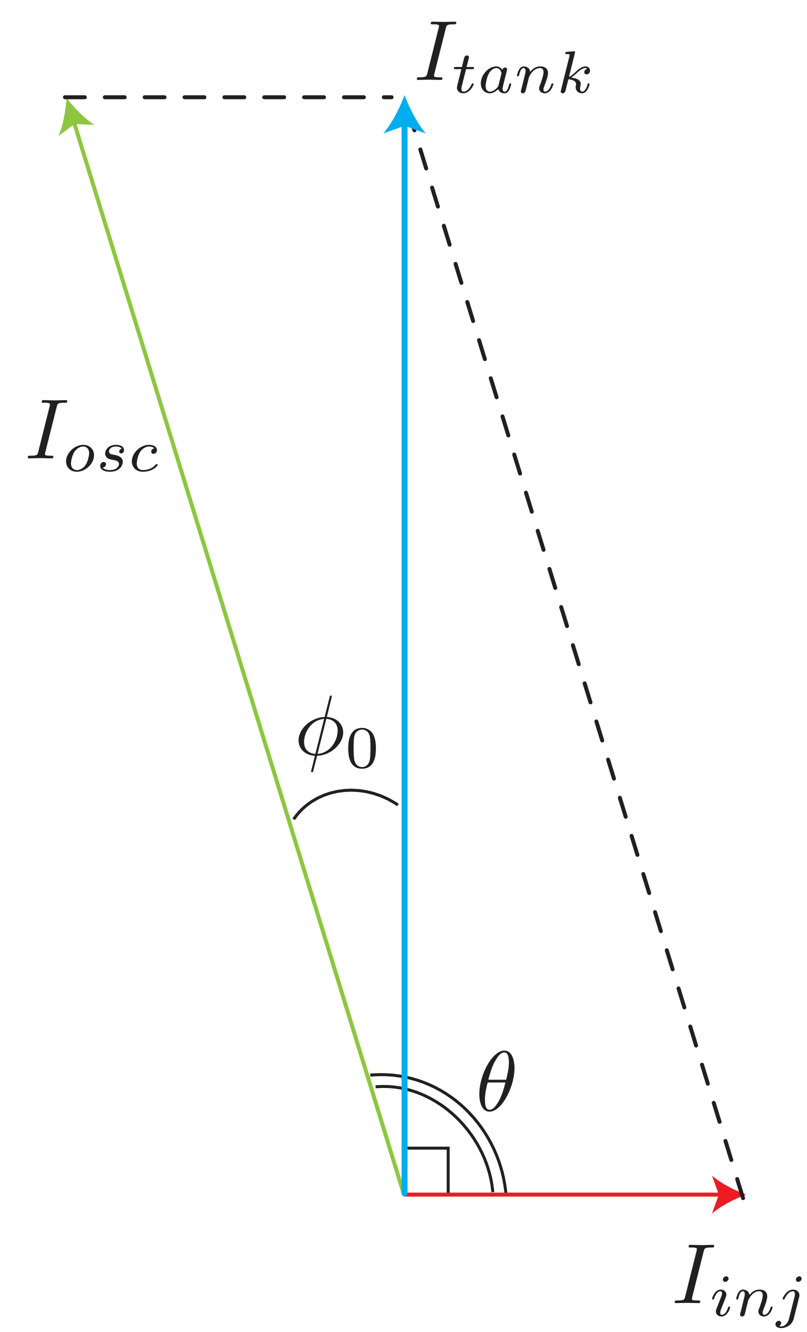
- At the edge of the lock range, the injected current is orthogonal to the tank current.
- The phase angle between the injected current and the oscillator is 90° + $\Phi_{0,max}$
- The lock range can be computed by noting that the tank phase shift is given by

$$\tan\phi_0 = \frac{2Q}{\omega_0}(\omega_0 - \omega_{inj})$$

$$\tan\phi_0 = \frac{2Q}{\omega_0}(\omega_0 - \omega_{inj}) = \frac{I_{inj}}{I_{tank}} = \frac{I_{inj}}{\sqrt{I_{osc}^2 - I_{inj}^2}}$$

$$(\omega_0 - \omega_{inj}) = \frac{\omega_0}{2Q}\frac{I_{inj}}{I_{osc}}\frac{1}{\sqrt{1 - \frac{I_{inj}^2}{I_{osc}^2}}}$$

Maximum Locking Range



Weak Injection Locking Range

- Adler first derived these results in a celebrate paper [Adler] under the conditions of a weak injection signal.
- The results for a large injection signal were first derived by [Paciorek] and then re-derived (as shown here) by [Razavi]
- Under weak injection:

$$I_{inj} \ll I_{osc}$$

$$\sin\phi_0 \approx \frac{I_{inj}}{I_{osc}} \sin(\theta)$$

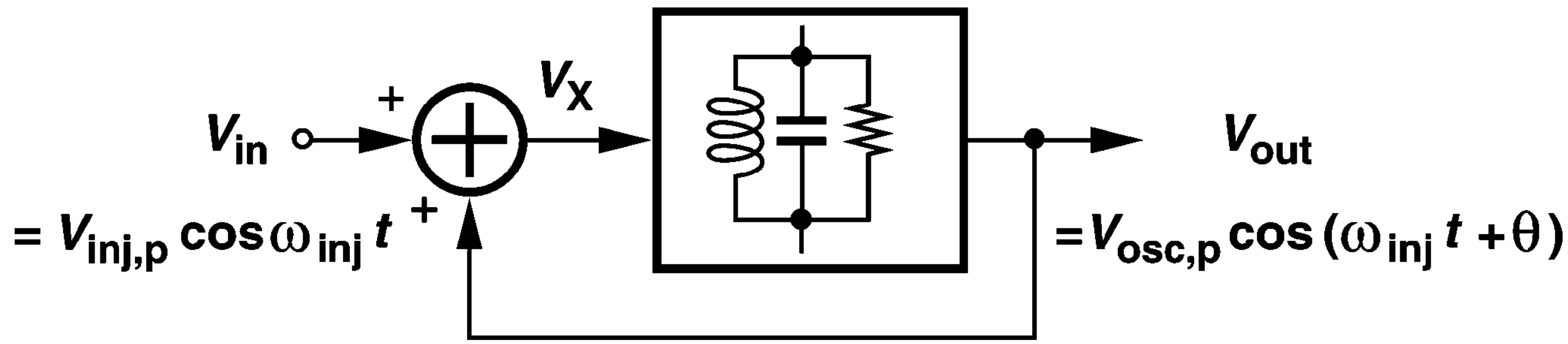
$$\sin\phi_0 = \frac{I_{inj}}{I_{tank}} \sin(\theta) \approx \tan\phi_0 = \frac{2Q}{\omega_0}(\omega_0 - \omega_{inj})$$

$$\sin(\theta) = \frac{2Q}{\omega_0} \frac{I_{tank}}{I_{inj}}(\omega_0 - \omega_{inj})$$

- At the edge of the locking range, the angle reaches 90°, and the injected signal is occurring at the peaks of the output, which cannot produce locking (think back to the Hajimiri phase noise model)

$$(\omega_0 - \omega_{inj}) = \frac{\omega_0}{2Q} \frac{I_{inj}}{I_{tank}}$$

Injection Pulling Dynamics



Source: [Razavi]

- Suppose now that the oscillator is under injection so that oscillator signal feeding the tank can be written as

$$V_X = V_{inj,p}\cos\omega_{inj}t + V_{osc,p}\cos(\omega_{inj}t+\theta)$$

$$V_X = (V_{inj,p} + V_{osc,p}\cos\theta)\cos\omega_{inj}t - V_{osc,p}\sin\theta\sin\omega_{inj}t$$

- This can be written as a cosine with a phase shift

$$V_X = \frac{V_{inj,p} + V_{osc,p}\cos\theta}{\cos\psi}\cos(\omega_{inj}t+\psi) \qquad \tan\psi = \frac{V_{osc,p}\sin\theta}{V_{inj,p} + V_{osc,p}\cos\theta}$$

- Which allows us to write the output as

$$V_{out} = \frac{V_{inj,p} + V_{osc,p}\cos\theta}{\cos\psi}\cos\left(\omega_{inj}t + \psi + \tan^{-1}\left[\frac{2Q}{\omega_0}\left(\omega_0 - \omega_{inj} - \frac{d\psi}{dt}\right)\right]\right)$$

Injection Pulling Dynamics [cont]

- We have assumed that the phase shift through the tank is given by the simple expression derived earlier where the instantaneous frequency is

$$\omega + \frac{d\psi}{dt}$$

$$\tan\alpha \approx \frac{2Q}{\omega_0}\left(\omega_0 - \omega - \frac{d\psi}{dt}\right)$$

- If the injection signal is weak, then the previous result simplifies to:

$$V_{out} = V_{osc,p}\cos\left(\omega_{inj}t + \psi + \tan^{-1}\left[\frac{2Q}{\omega_0}\left(\omega_0 - \omega_{inj} - \frac{d\psi}{dt}\right)\right]\right)$$

- Which must equal to the output voltage, or the phase shifts must equal

$$\psi + \tan^{-1}\left[\frac{2Q}{\omega_0}\left(\omega_0 - \omega_{inj} - \frac{d\psi}{dt}\right)\right] = \theta$$

Pulling [cont]

- These equations can be manipulated into a form of Adler's equation:

$$\frac{d\theta}{dt} = \omega_0 - \omega_{inj} - \frac{\omega_0}{2Q}\frac{V_{inj,p}}{V_{osc,p}}\sin\theta$$

$$\frac{d\theta}{dt} = \omega_0 - \omega_{inj} - \omega_L \sin\theta \qquad \omega_L \triangleq \frac{\omega_0}{2Q}\frac{V_{inj,p}}{V_{osc,p}}$$

- This equation describes the dynamics of the phase change of the oscillator under injecting pulling. If we set the derivative to zero, we obtain the injection locking conditions (same as before).
- Notice that maximum value of the rhs is quite small, which means that the rate of change of phase is
- This equation can be used to study the behavior of locking signals outside the lock range. Note that this equation agrees with our graphical analysis:

$$\frac{d\theta}{dt} = 0$$

Pull-In Process

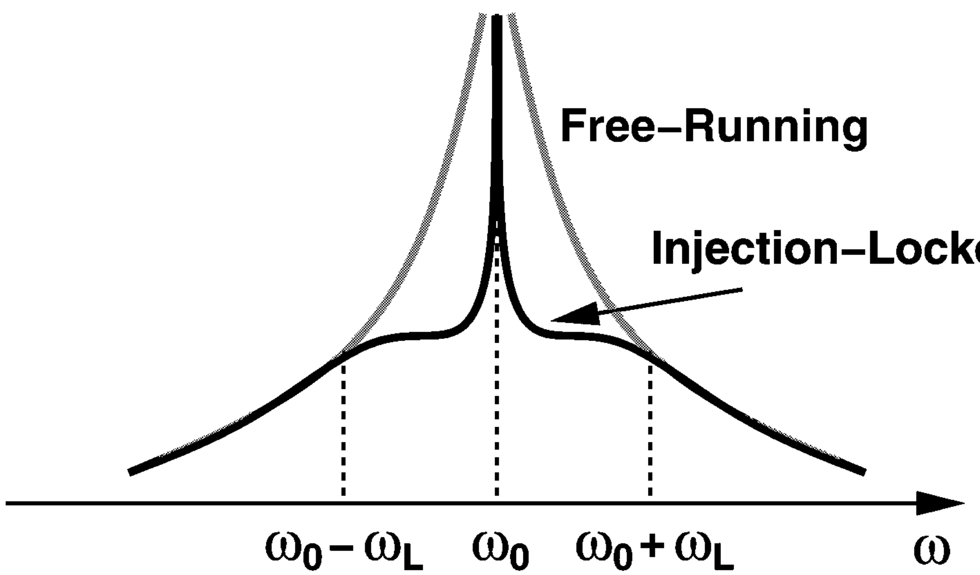
$$\theta(t) = 2\tan^{-1}\left[\frac{1}{\sin\theta_0} - \cot\theta_0 \tanh\left(\frac{\omega_L \cos\theta_0}{2}(t - t_0)\right)\right]$$

$$\theta_0 = \sin^{-1}\frac{\omega_0 - \omega_1}{\omega_L}$$

- θ_0 is the steady-state phase shift between the injected signal and the active device signal and t_0 is an integration constant that depends on the initial phase shift.
- From this equation the lock-in time can be computed (it's approximately an exponential process):

$$t_L = \frac{2}{\omega_L \cos\theta_0}\tanh^{-1}\left[\frac{1 - \sin\theta_0 \tan\left(\frac{\theta_L}{2}\right)}{\cos\theta_0}\right] + t_0$$

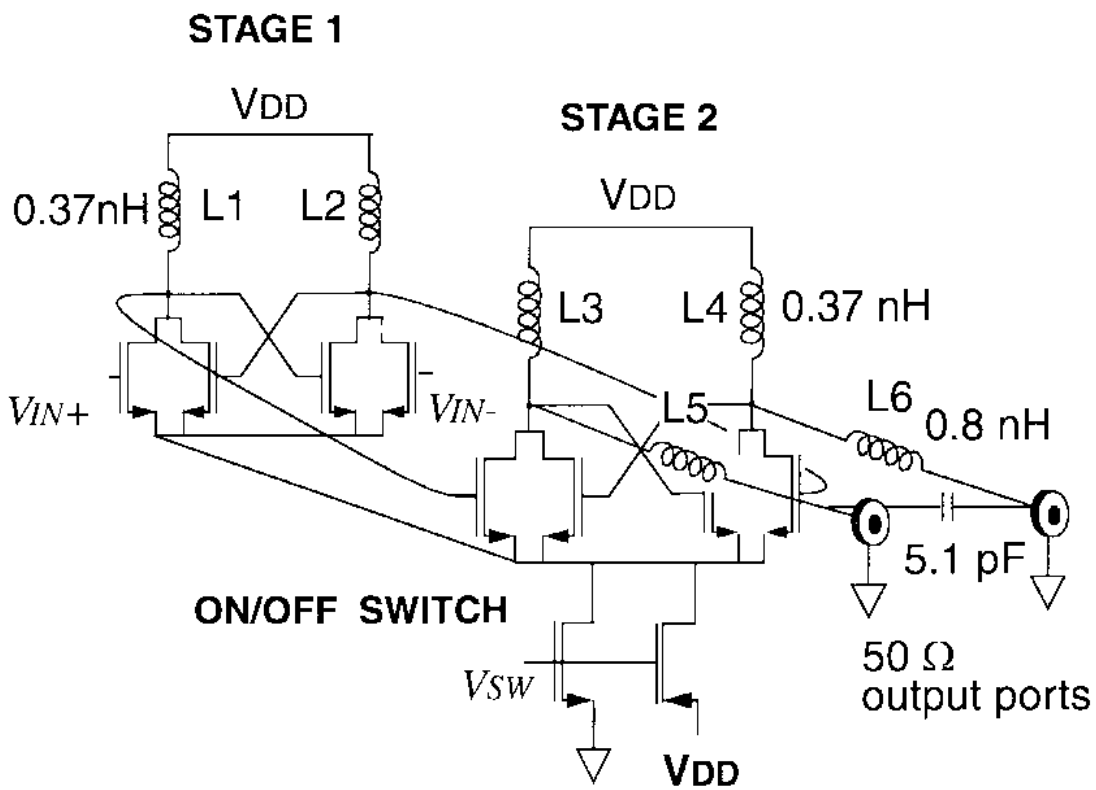
Phase Noise in Injection Locked Systems

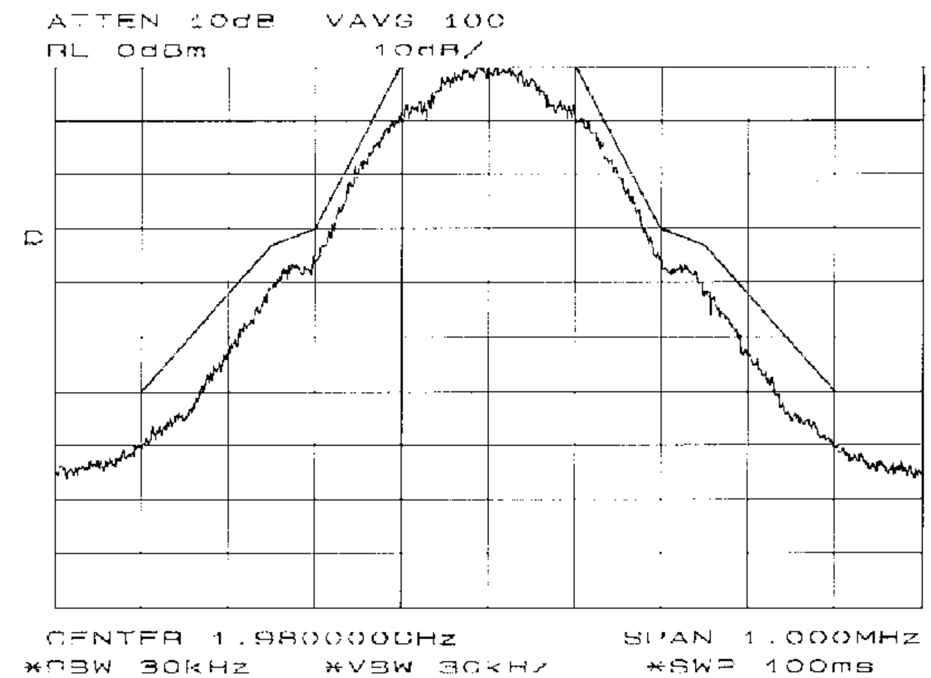


Source: [Razavi]

- Under a lock, the phase of the oscillator follows the phase of the injection signal. If a "clean" signal is used to lock a VCO, then the phase noise would improve up to the locking range.
- At the edge of the lock, the injected signal cannot correct for the phase noise since it injects energy at a 90° phase offset, where the signal has a peak amplitude

Example: GSM Class E PA





Source: [Tsai]

TRANSISTOR SIZING

STAGE 1	STAGE 2	ON/OFF SWITCH
Input: 980/0.35	Input: 3600/0.35	NMOS: 31580/0.35
Assist: 980/0.35	Assist: 4800/0.35	PMOS: 500/0.35

- Large output stage device is hard to drive (large capacitance).
- Use the PA to drive itself! That's an oscillator, right? Yes.
- Use injection locking to inject phase modulation. Need "off" switch to turn off transmission.

Example: Low Power Transmitter

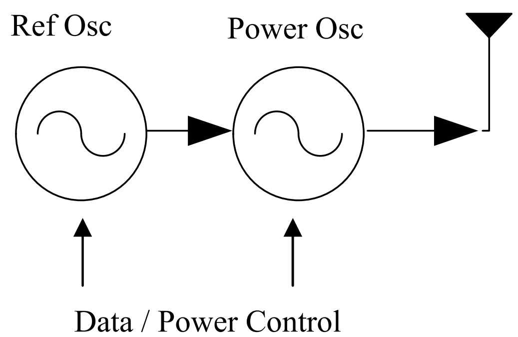
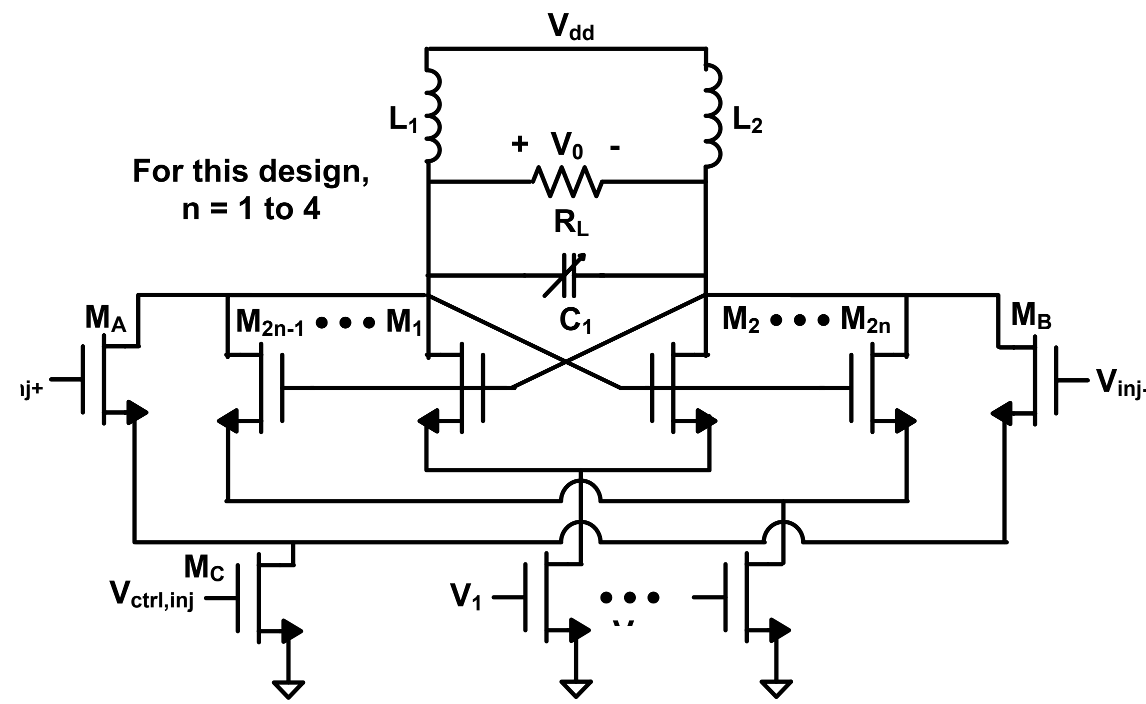


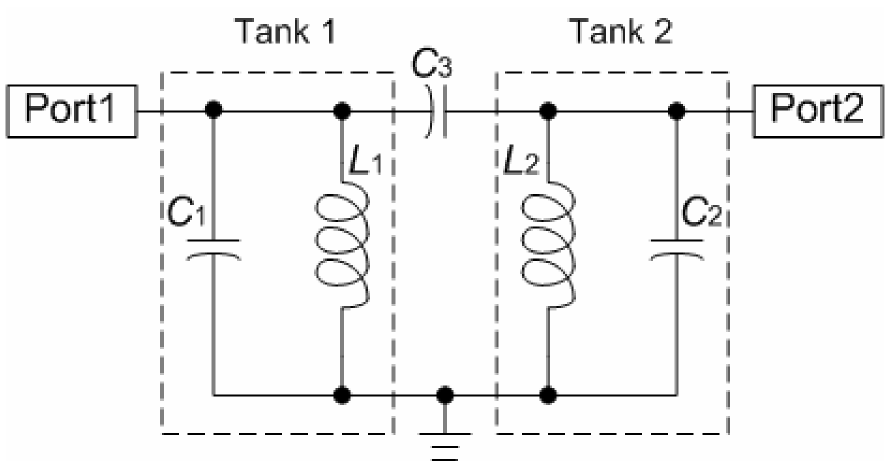
Fig. 4.9: Output spectrum when power oscillator is (left) free running (right) locked.



Source: [Chee]

- In a low-power radio, the overall transmitter efficiency is very important. The PA output power is modest (~1mW), but since the overall efficiency should be large, the entire transmitter should not consume more than 2-3mW.
- By using injection locking one can reduce the power consumption of the driver stages and end up with a minimal transmitter architecture.

Example: Dual Mode VCO



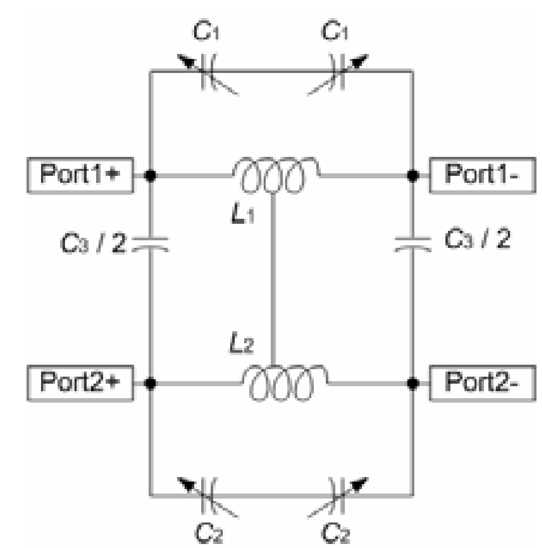
$$\omega_L = \sqrt{\frac{K_2 - \sqrt{\Delta}}{2K_3}} \qquad \omega_H = \sqrt{\frac{K_2 + \sqrt{\Delta}}{2K_3}}$$

$$\Delta = \left(\frac{C_3}{L_1} - \frac{C_3}{L_2} - \frac{C_1}{L_2} + \frac{C_2}{L_1}\right)^2 + 4\frac{C_3^2}{L_1 L_2}$$

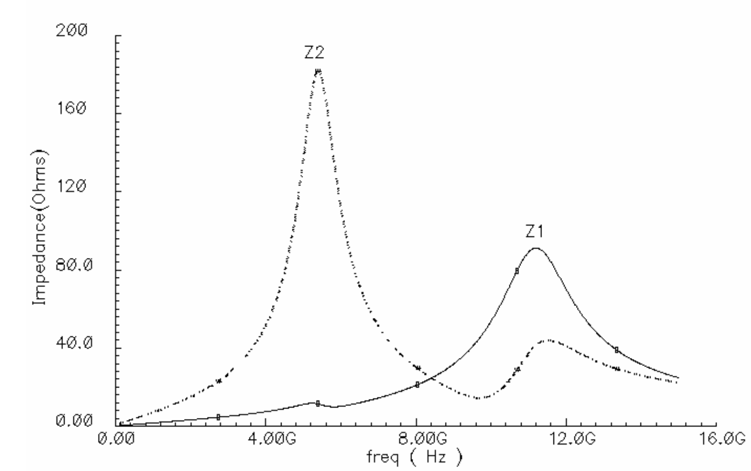
$$K_2 = \frac{C_3}{L_1} + \frac{C_3}{L_2} + \frac{C_1}{L_2} + \frac{C_2}{L_1}$$

$$K_3 = C_3 C_1 + C_3 C_2 + C_1 C_2$$

- A coupled tank has two resonant modes. From each port of the oscillator, one mode is dominant.
- In a fully differential version shown to the right, the impedance variation with frequency is shown.
- Can we build an oscillator that sustains both modes?

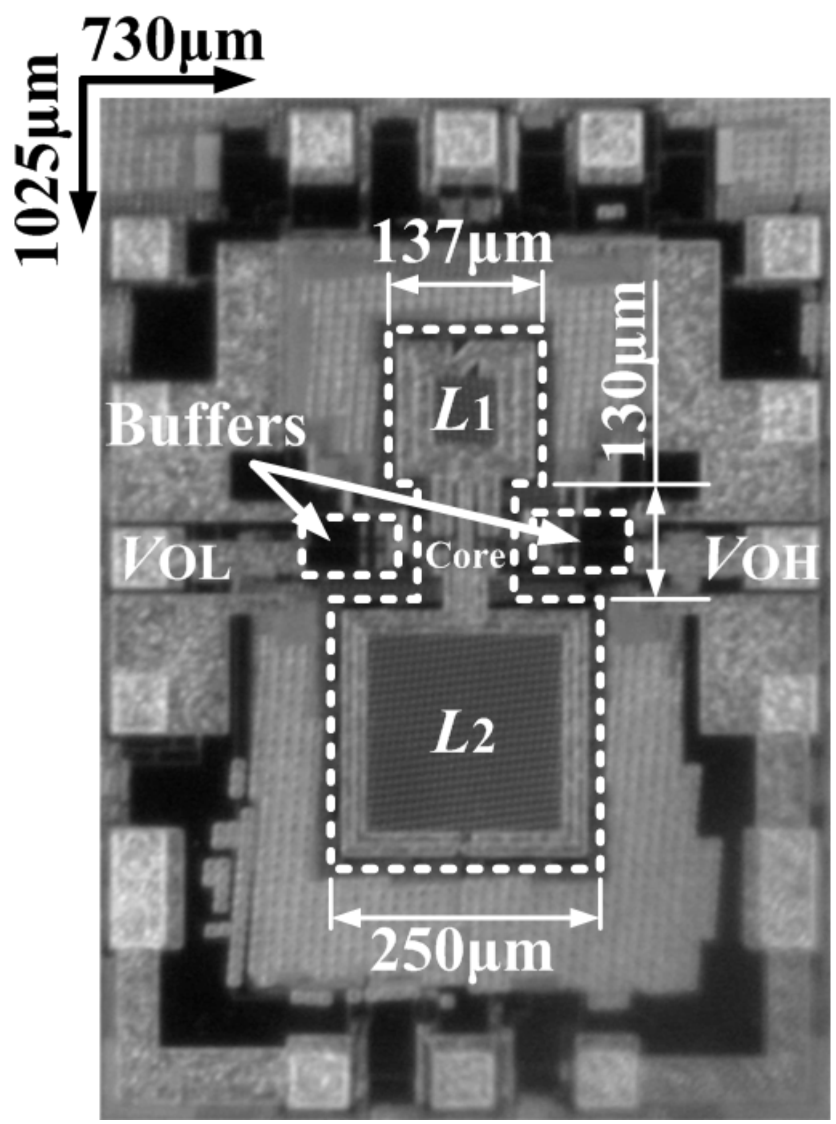


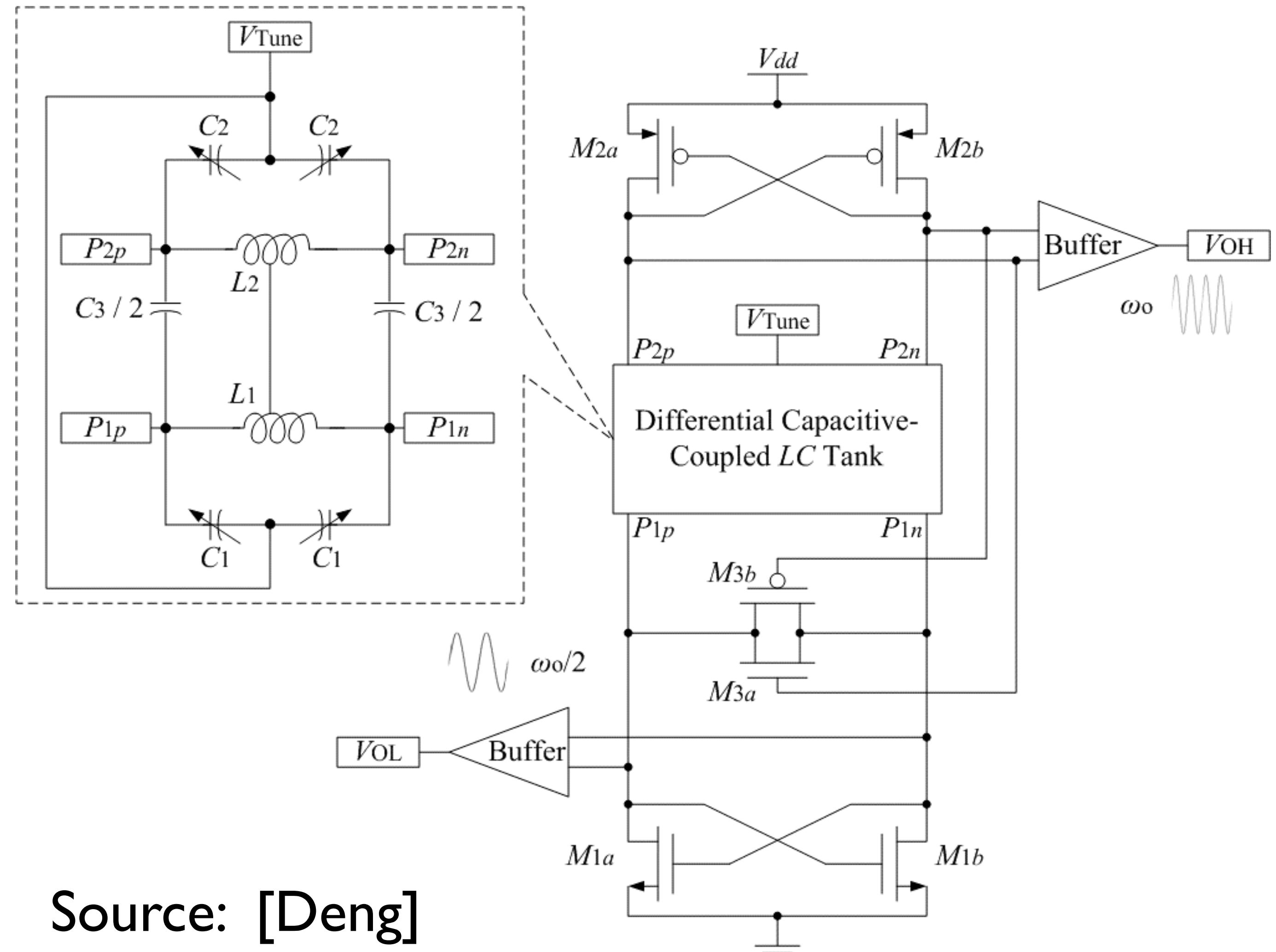
(a)



(b)

Dual Mode VCO (cont)

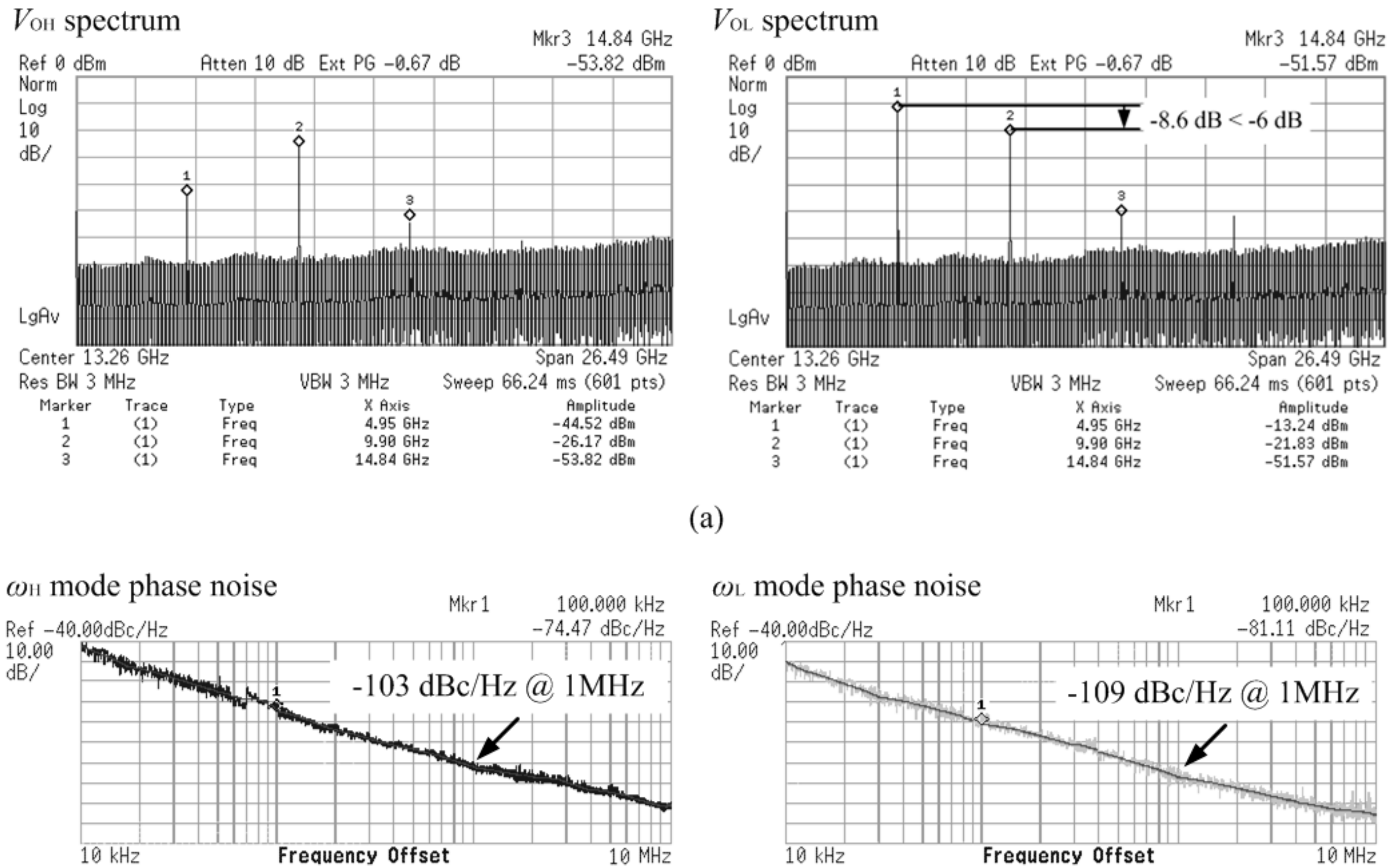




Source: [Deng]

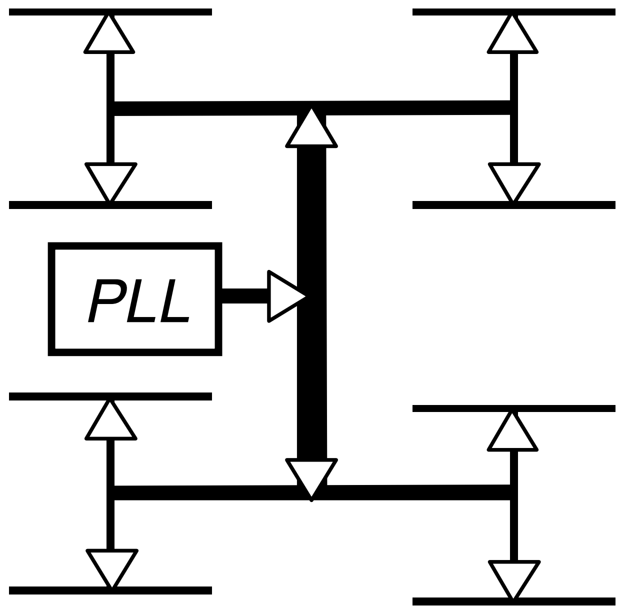
- Two cross-coupled pairs are used to sustain each mode by providing sufficient negative resistance. The PMOS side is running at a lower frequency whereas the NMOS side runs at the higher frequency.
- Transistors M3a/M3b are used for injection locking.

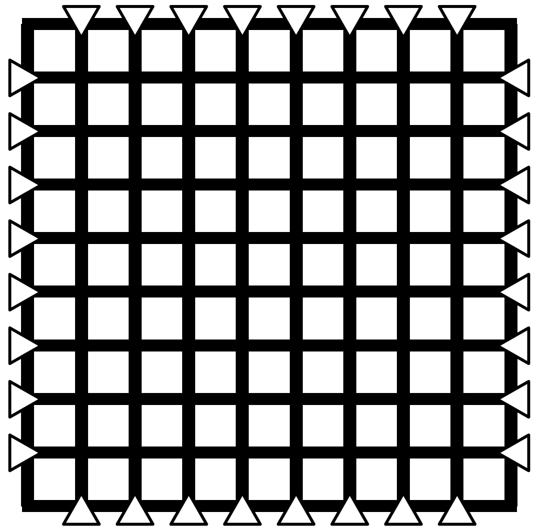
Dual Mode VCO Spectrum



- Without injection locking, each mode runs independently. The frequencies are not integrally related, so the unlocked spectrum contains not only the two modes and harmonics, but also every intermodulation component.
- When the modes are locked, the intermodulation components disappear. The high frequency VCO is divided by two in this configuration.
- More on injection locking dividers to come...

Clock Distribution

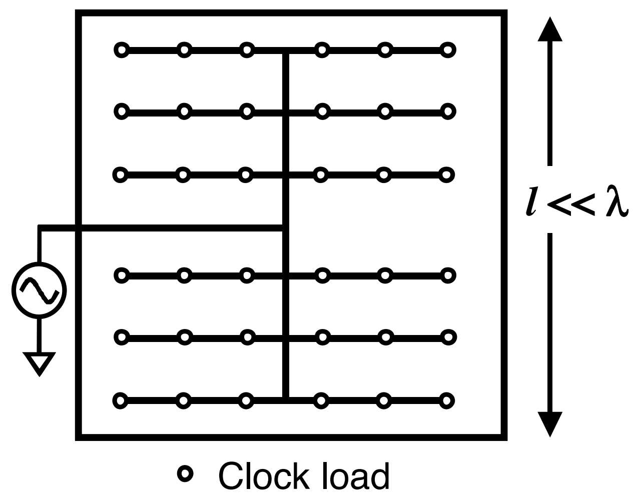


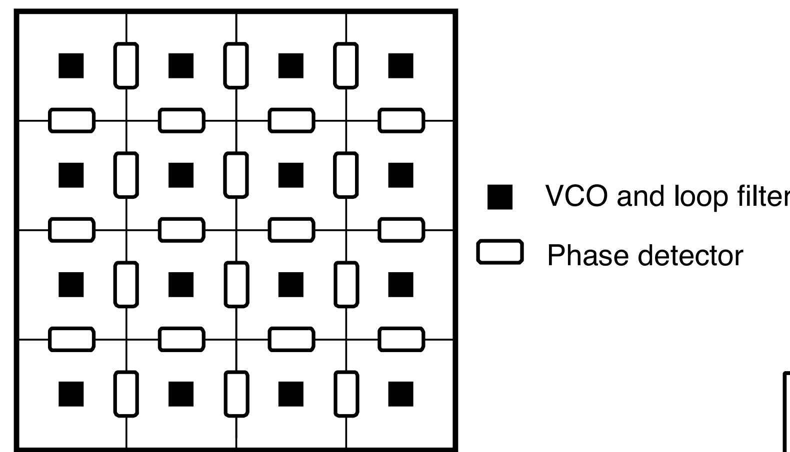


Source: [Mahony]

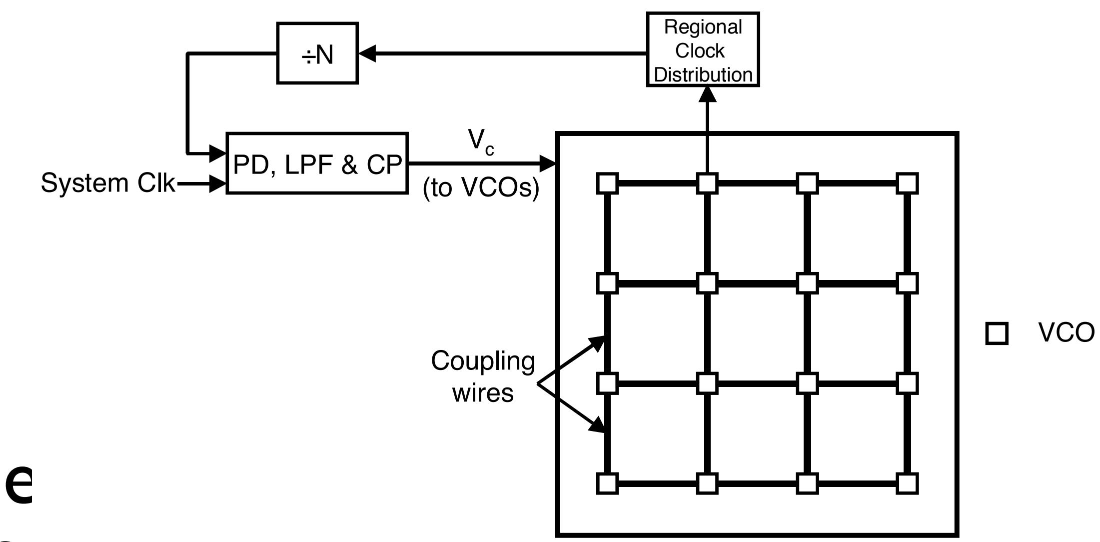
- Delivering a clock in a large chip is a major challenge and a big source of power consumption. Figures of merit include skew, jitter, and power.
- A buffered H-tree or a grid are common approaches. The grid has skew and larger capacitance.

Clock Distribution: Distributed



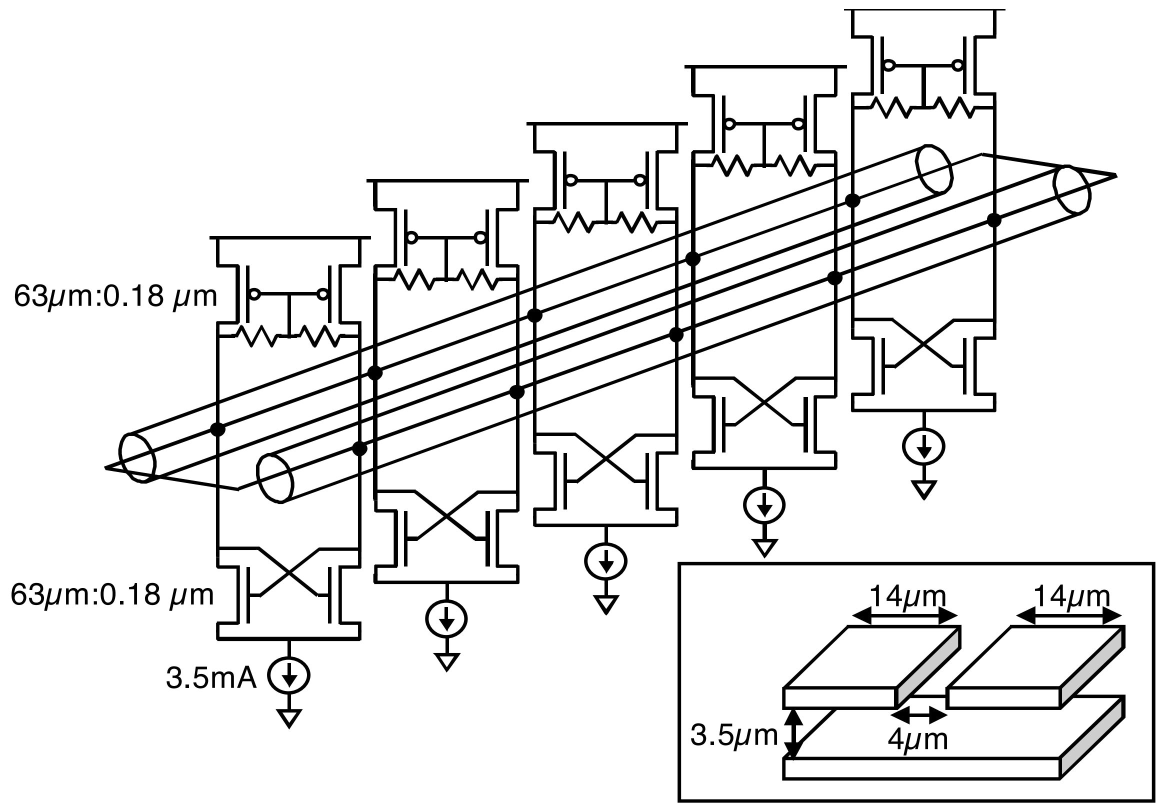


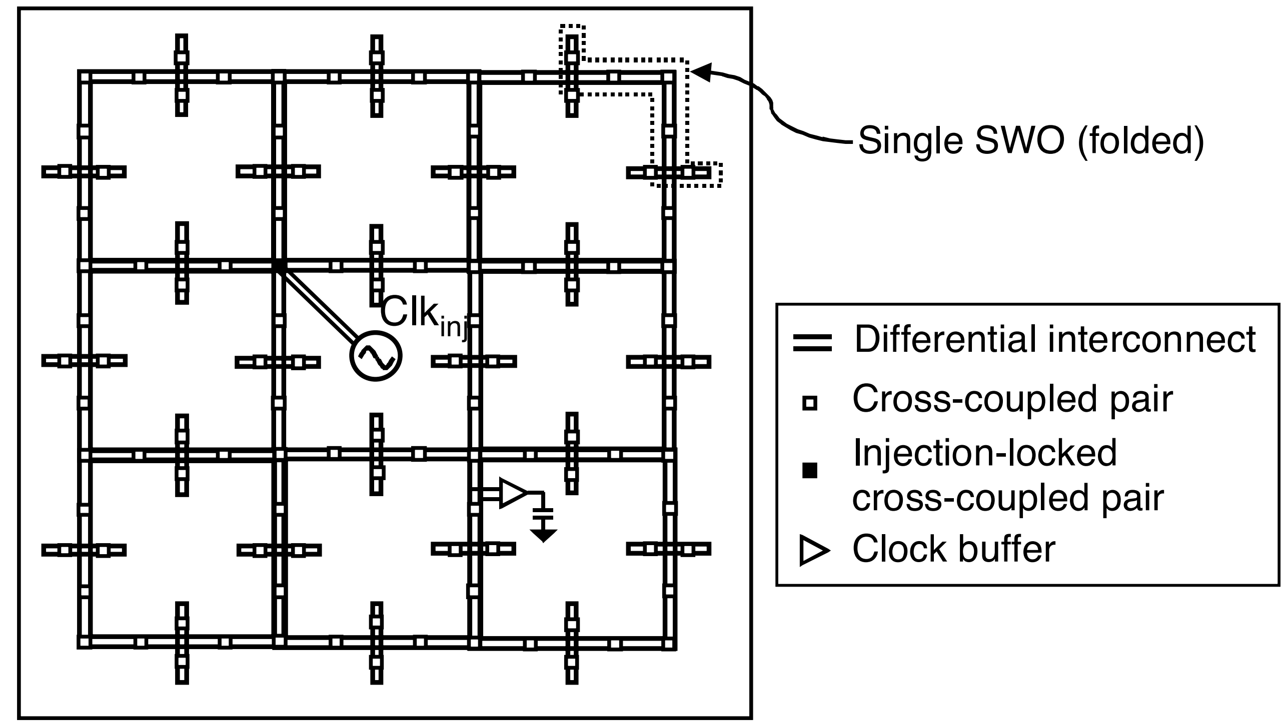
Source: [Mahony]

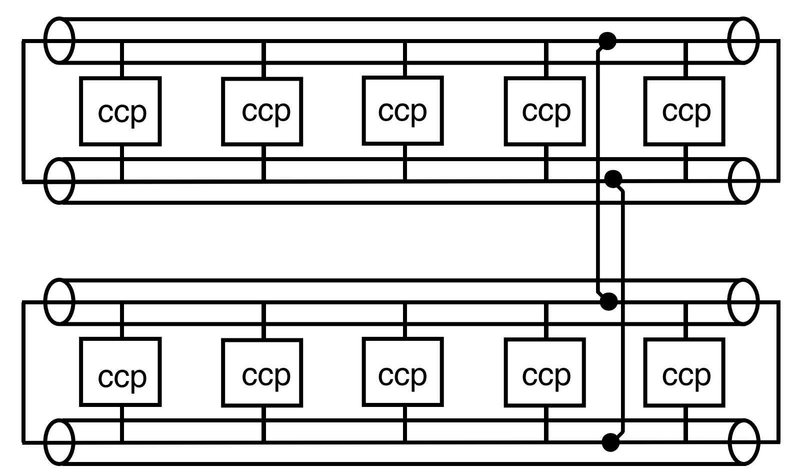


- Standing wave oscillators tune out the capacitance of the line and form a resonant network.
- A distributed approach locks an array of VCO's to it's neighbors.
- The neighbors can also be injection locked to one another to eliminate the phase detectors.

Standing Wave Oscillators (SWO)



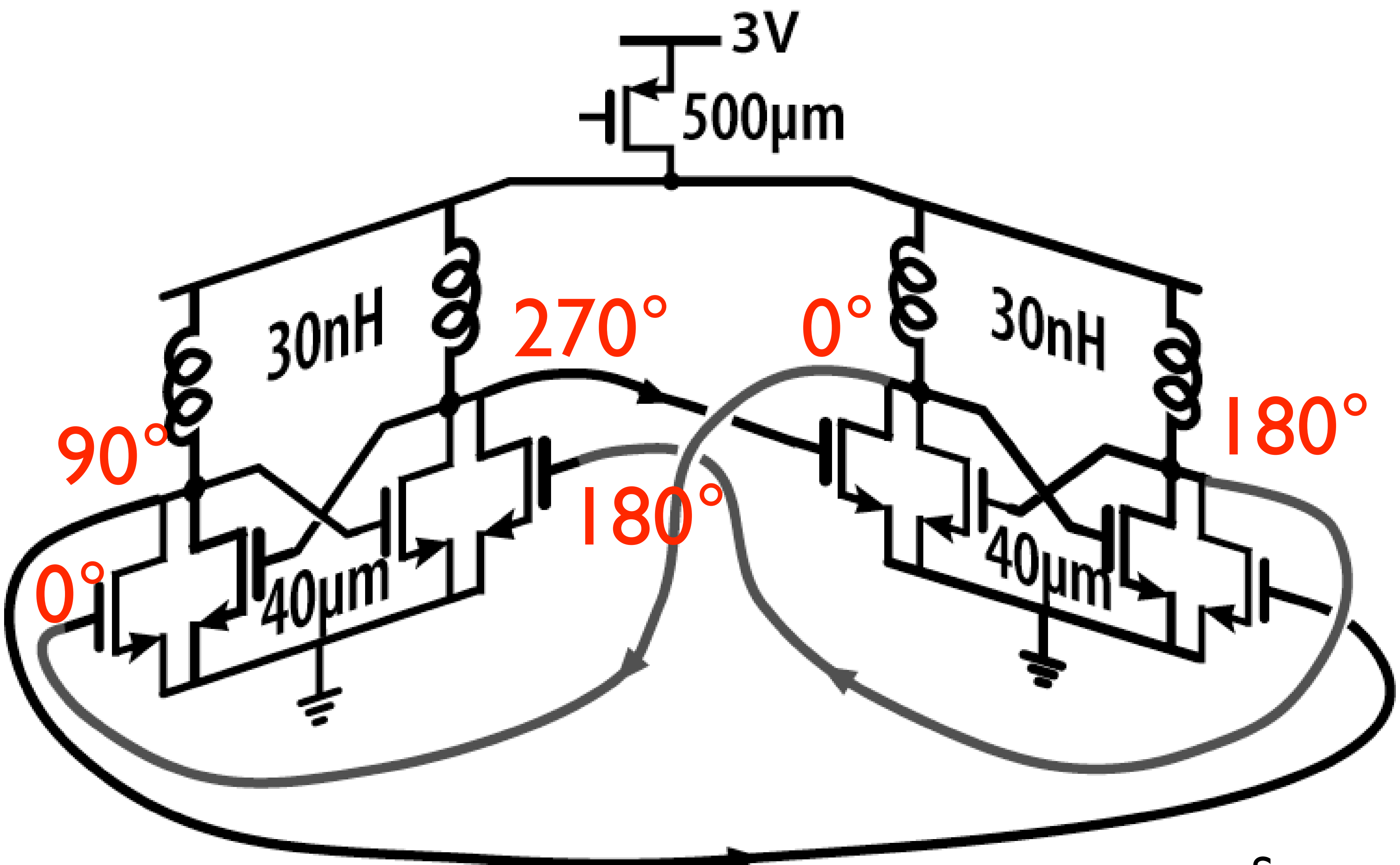




- The negative resistance to sustain oscillation is distributed along the line.
- One disadvantage is that the amplitude of the clock varies along the line.
- SWO's can be injection locked together to form larger clock trees.

Source: [Mahony]

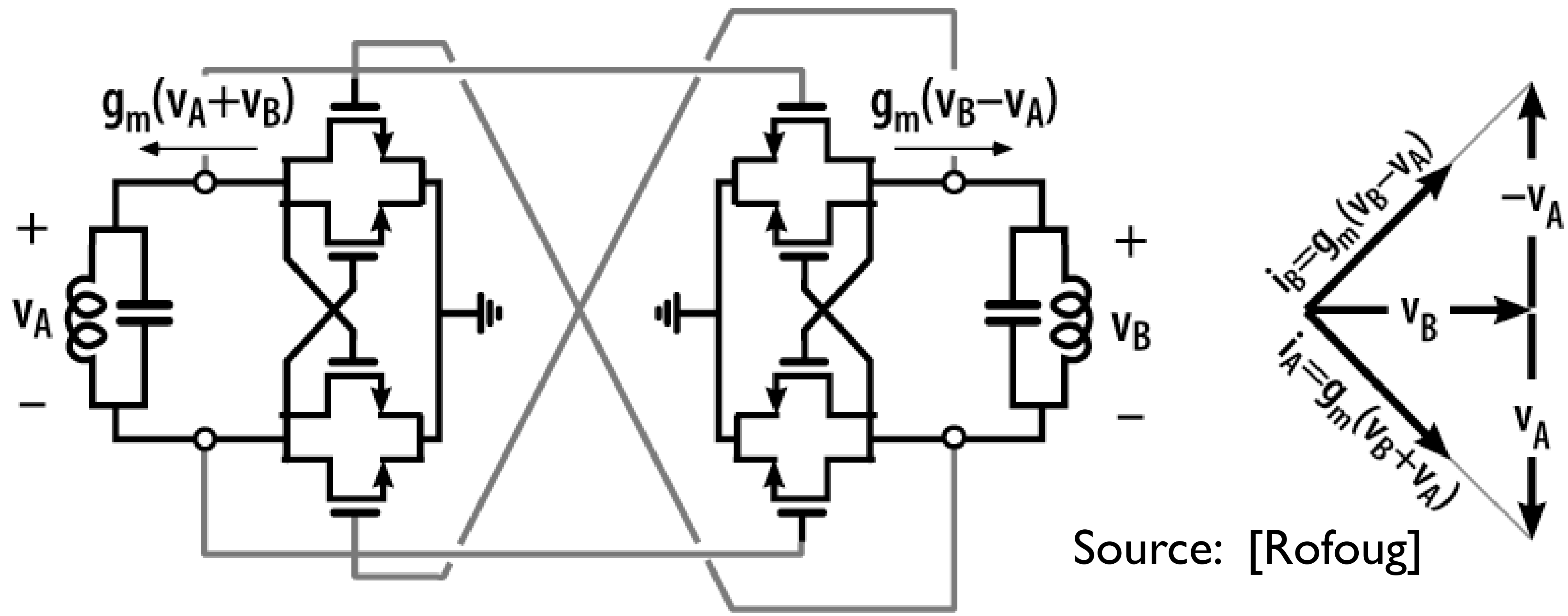
Quadrature Locked VCOs (QVCO)



Source: [Rofoug]

- Two VCO's are coupled together using extra transistors as shown. If the oscillators are identical, we expect that the amplitude and frequency of oscillation should be identical.
- Because of the phase of the coupling, it can be shown that they lock in quadrature...

QVCO: Quadrature Lock



Source: [Rofoug]

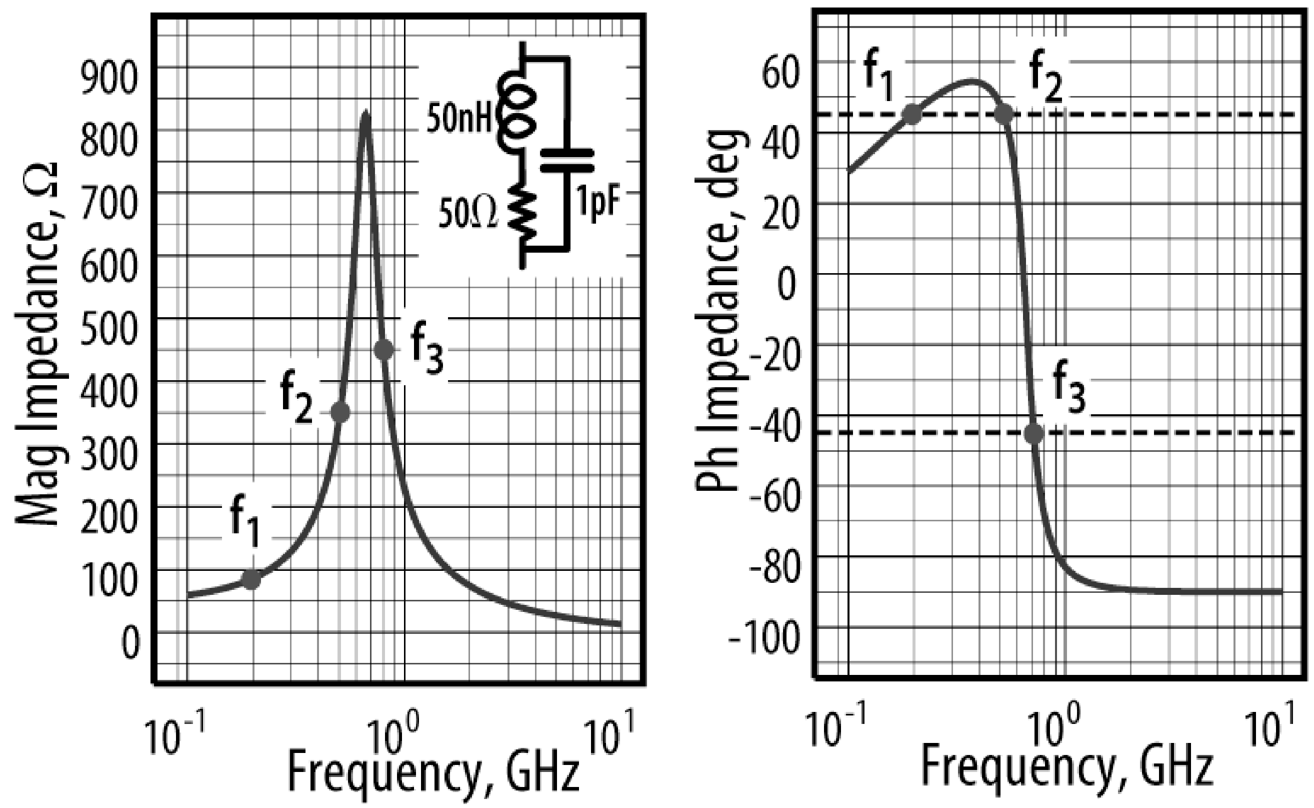
- Note that the tank "A" is driven by the tank voltage VA plus the tank voltage of "B".
- The right tank, though, is driven by voltage V_B but voltage $-V_A$.
- Assuming that the LC tanks and FETs are identical, then the phasor currents flowing into the tanks must have equal magnitude:

$$|v_A + v_B| = |-v_A + V_B|$$

$$|1 + e^{j\theta}| = |1 - e^{j\theta}|$$

$$\cos\theta = 0 \rightarrow \theta = \pm 90^\circ$$

QVCO: Lead/Lag Ambiguity

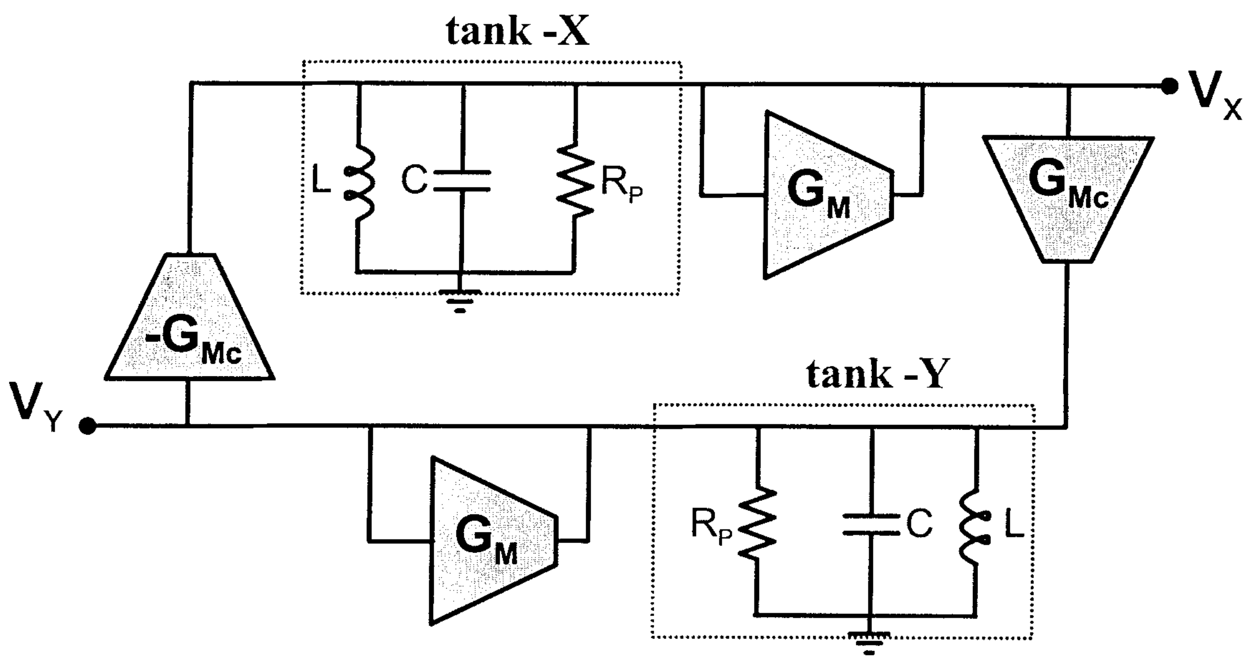


Source: [Rofoug]

- Note that the tank current/voltage has a 90° relation, or the phase of the impedance is ±45°. Three frequencies satisfy this constraint, but f_3 has the highest loop gain.
- It's clear that there are two possible solutions: lead or lag in phase between "A" and "B".
- In any real oscillator, the two oscillators are not perfectly symmetric, and it can be shown that there is only one unique solution (where the loop gain is highest and the net phase shift around the loop is 0°)

QVCO Loop Gain

- Linear model of QVCO is shown. The loop gain is easily computed and the frequency at which the loop gain is zero is the oscillation frequency.
- Notice that the oscillation is off resonance. This results in lower phase noise.



Source: [Andreani]

$$G_{\text{loop}}(s) = -G_{\text{Mc}}^2 \left(\frac{sL}{1 + sL(1/R_P - G_M) + s^2\text{LC}} \right)^2$$

$$\frac{G_M}{C} = \frac{1}{CR_P} \cong \frac{\omega_0}{Q}$$

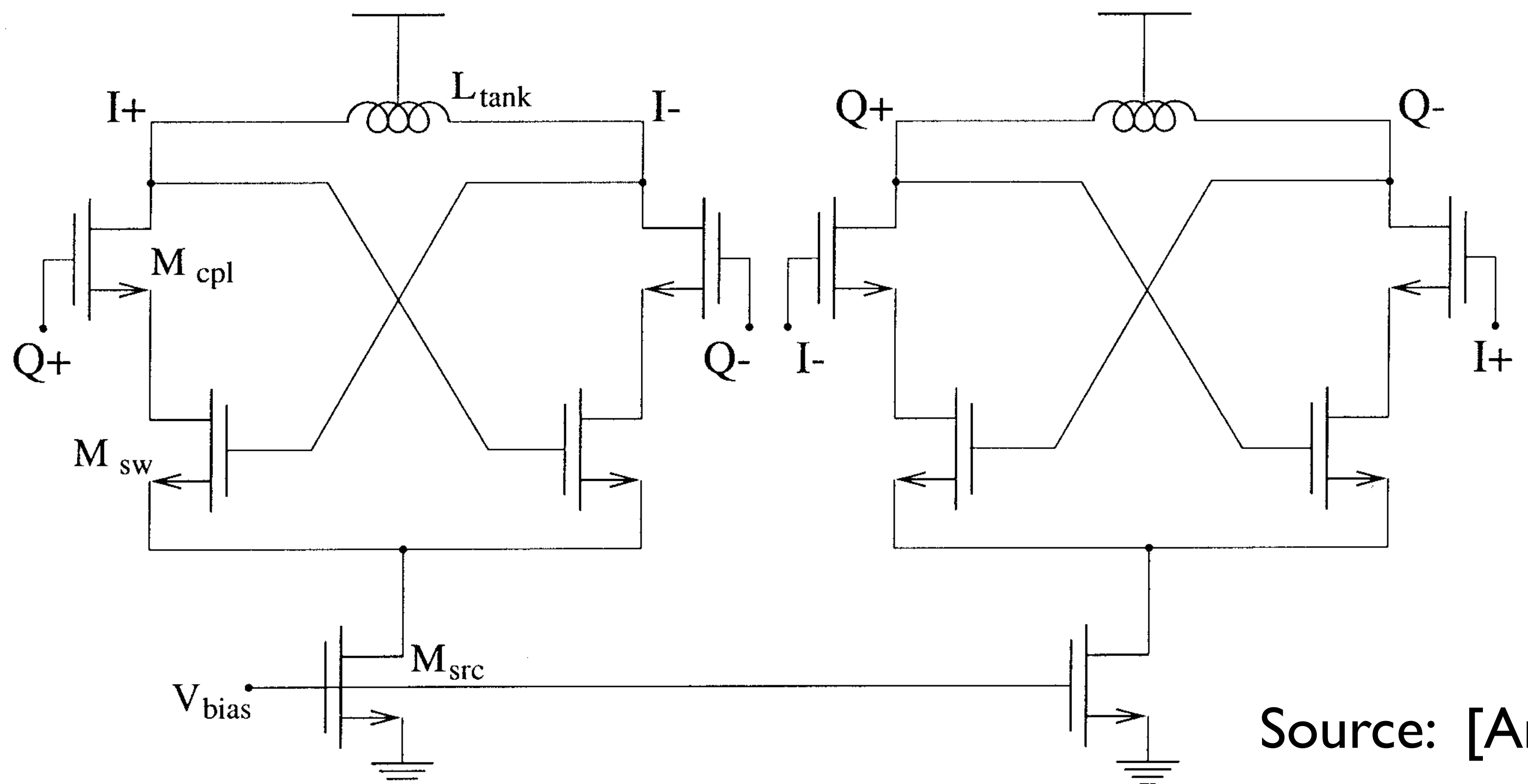
$$G_M L = \frac{L}{R_P} \cong \frac{1}{\omega_0 Q}$$

$$\omega_1 = \sqrt{\frac{LG_{\text{Mc}}^2 + 2C + \sqrt{L^2 G_{\text{Mc}}^4 + 4LCG_{Mc}^2}}{2LC^2}}$$

$$\omega_2 = \sqrt{\frac{LG_{\text{Mc}}^2 + 2C - \sqrt{L^2 G_{\text{Mc}}^4 + 4LCG_{\text{Mc}}^2}}{2LC^2}}$$

$$\omega_1 \cong \omega_0 + \frac{G_{\text{Mc}}}{2C}, \qquad \omega_2 \cong \omega_0 - \frac{G_{\text{Mc}}}{2C}$$

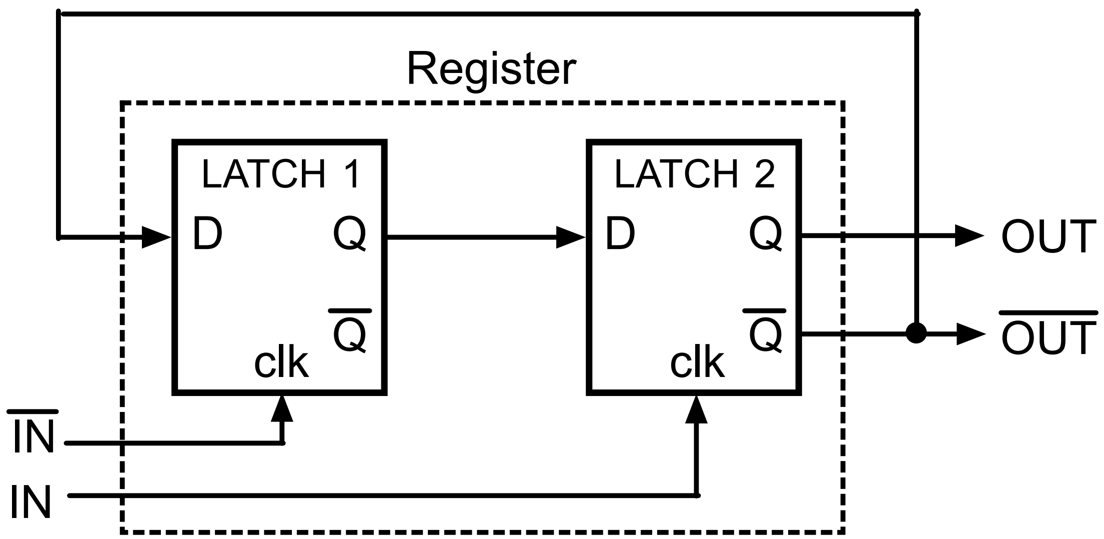
QVCO in the Literature

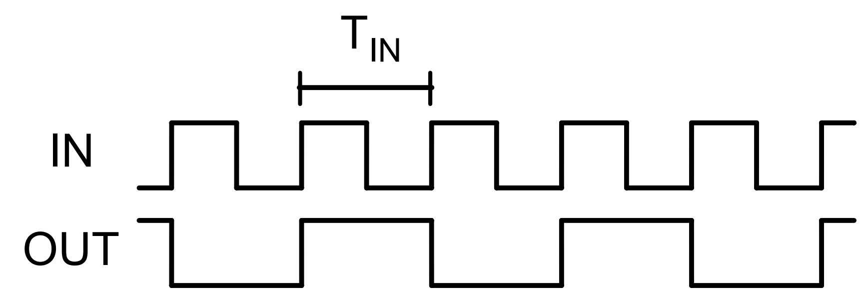


Source: [Andreani]

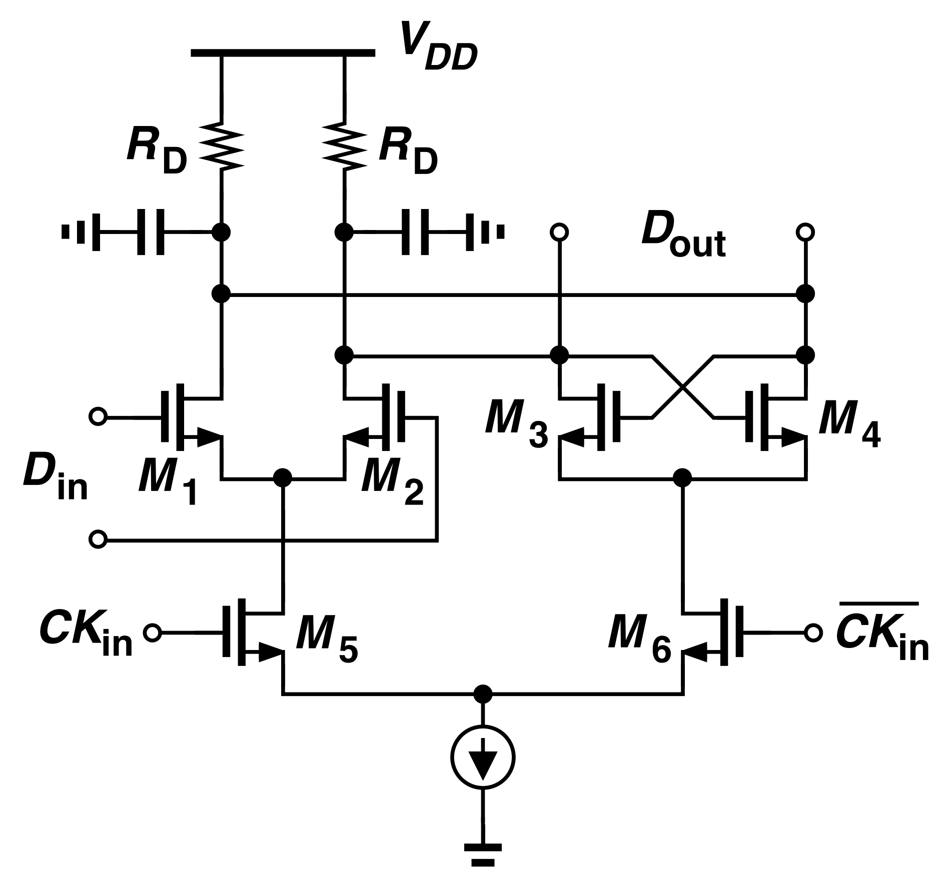
- Several modifications to the QVCO have been proposed for improved performance
- In particular, it has been shown that if a weaker coupling is introduced, the phase noise improves (the circuit locks closer to the tank resonance), but at the cost of imperfect quadrature generation
- A series connected quadrature generation scheme proposed by [Andreani] has better phase noise performance.

Static Frequency Dividers

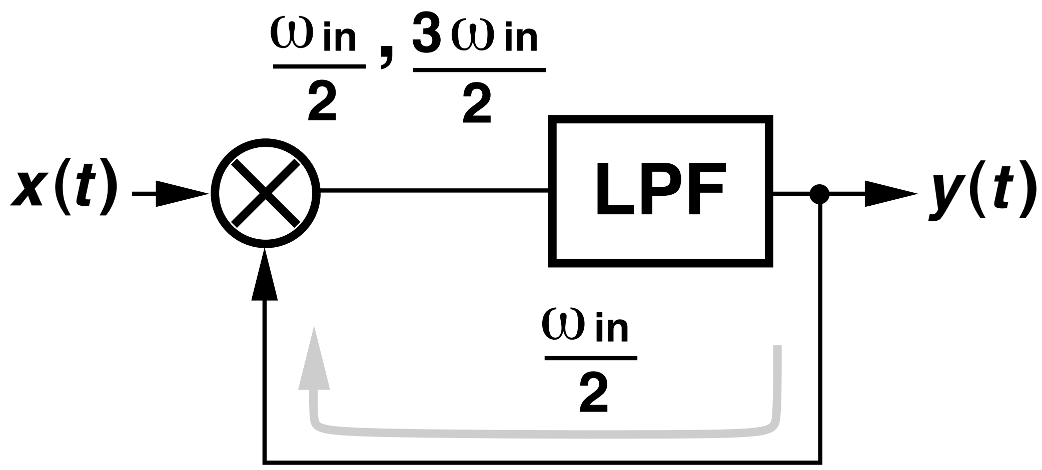




- This is a simple divider structure uses a master/slave topology.
- Two latches are cascaded into a negative feedback loop (the output will therefore toggle).
- Since two clock cycles are required to pass the data from one latch to the next, it naturally divides by 2.



Miller Divider (Regenerative)

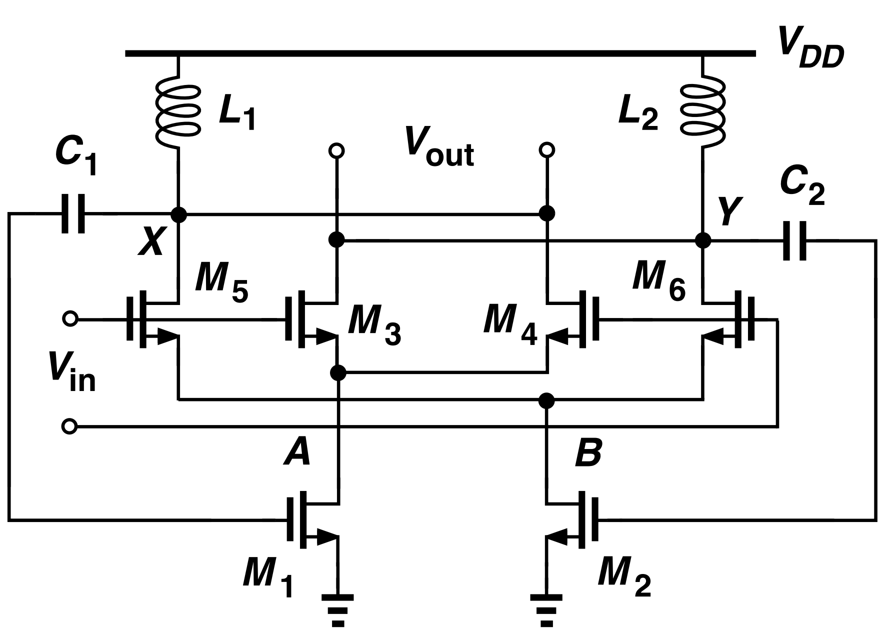


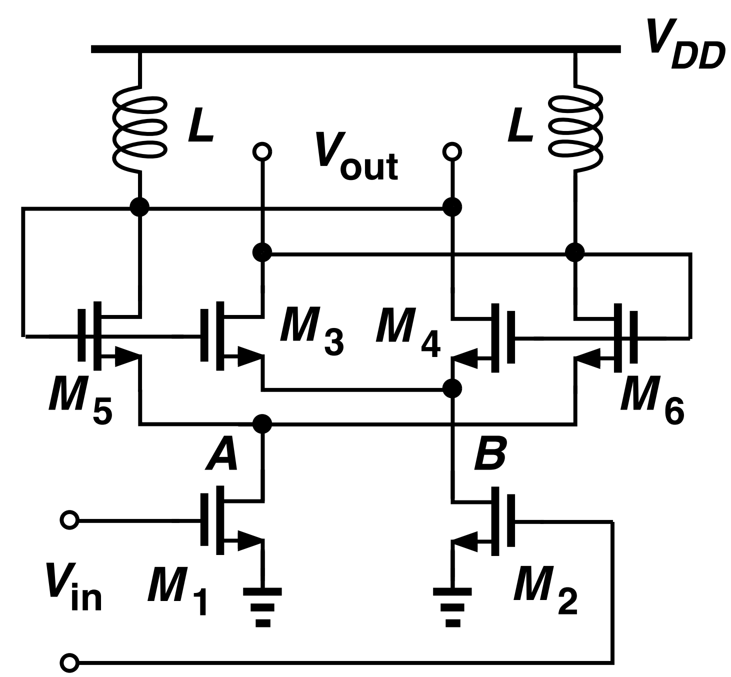
- The output signal of the feedback loop is mixed with the input signal. The output of the (ideal) mixer has the sum and difference components.
- Only the difference component is amplified (due to the LPF) and gained up.
- If the loop gain is greater than one and the loop phase is zero degrees, the system regenerates the input.
- The output frequency in steady state must therefore satisfy:

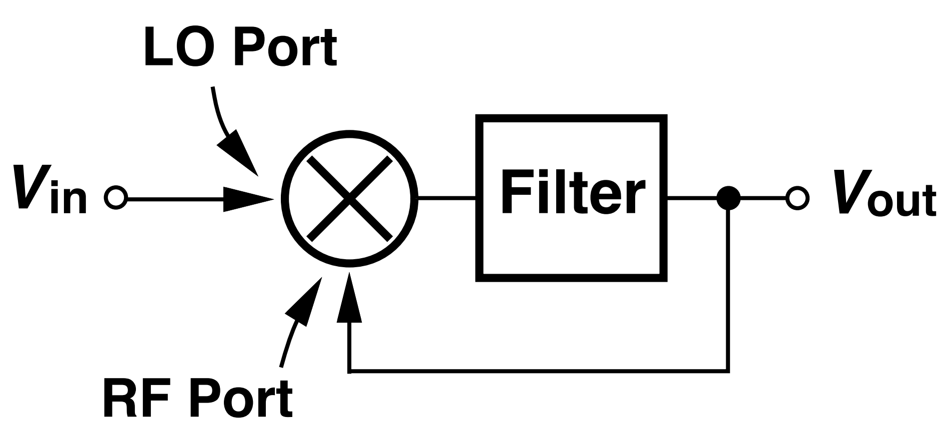
$$f_{out} = f_{in} - f_{out}$$

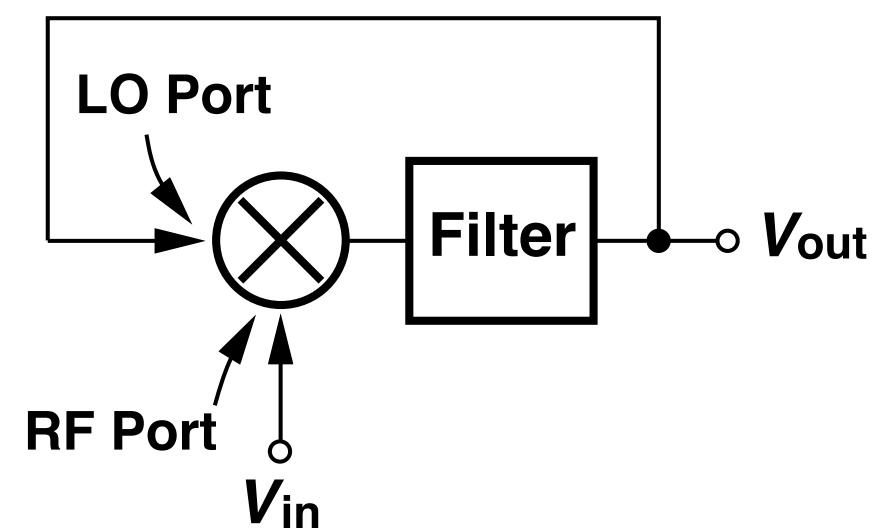
$$f_{out} = \frac{1}{2} f_{in}$$

Miller Divider Circuit Details



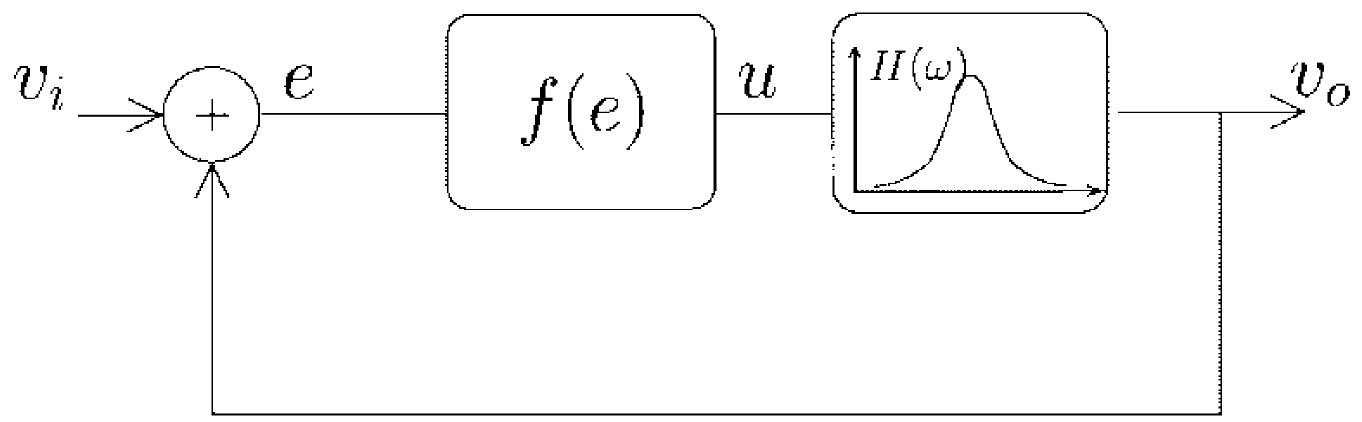






- Use a double-balanced Gilbert cell mixer to realize divider

Injection Locked Dividers [IL-Div]



- An injection locked divider is nothing but an injection locked system where the input frequency is at a harmonic of the free-running frequency of the oscillator.
- Model the system as a non-linearity *f*(e) and a bandpass transfer function *H*(ω).
- Assume that the free-running loop has a stable oscillation frequency. The system is injection locked to a super-harmonic of the free-running frequency.
- The non-linearity in the loop must create intermodulation products that fall in the passband of the loop.

IL-Div Analysis

- Suppose the injection signal is a sinusoidal signal which is added to the oscillator's signal (at a sub-harmonic of the injection). The non-linearity acts on both signals and is filtered by the RLC circuit:

$$v_i(t) = V_i \cos(\omega_i t + \phi) \qquad v_o(t) = V_o \cos(\omega_o t)$$

$$u(t) = f(e(t)) = f(v_o(t) + v_i(t)) \qquad H(\omega) = \frac{H_0}{1 + j2Q\frac{\omega - \omega_r}{\omega_r}}$$

- It can be shown that if the output signal contains various harmonics and intermodulation terms which can be written as:

$$u(t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} K_{m,n} \cos(m\omega_i t + m\phi) \cos(n\omega_0 t)$$

- where $K_{m,n}$ is the intermodulation component of $f(v_i + v_o)$

IL-Div Fourier Analysis

- To see this, write the signals in the following form:

$$v_i = V_i \cos(\beta)$$

$$v_o = V_o \cos(\alpha)$$

$$f(e) = f(v_i + v_o)$$

- which means that f is periodic in α and β. For every β define a periodic function g(α) as follows

$$g(\alpha) = f(v_o + V_i \cos(\alpha))$$

- Note that:

$$g(\alpha + 2\pi) = g(\alpha)$$

$$g(-\alpha) = g(\alpha)$$

- So that we can write:

$$g(\alpha) = \sum_{m=0}^{\infty} L_m(\beta)\cos(m\alpha)$$

$$L_0(\beta) = \frac{1}{2\pi}\int_0^{2\pi} f(V_o\cos(\beta) + V_i\cos(\alpha))d\alpha$$

$$L_m(\beta) = \frac{1}{\pi}\int_0^{2\pi} f(V_o\cos(\beta) + V_i\cos(\alpha))\cos(m\alpha)d\alpha$$

IL-Div [Fourier Analysis cont.]

- But since L_m is a periodic and even function of β we can write:

$$L_m(\beta) = \sum_{n=0}^{\infty} K_{m,n} \cos(n\beta)$$

$$K_{m,0} = \frac{1}{2\pi} \int_0^{2\pi} L_m(\beta) d\beta$$

$$K_{m,n} = \frac{1}{\pi} \int_0^{2\pi} L_m(\beta) \cos(n\beta) d\beta$$

- which results in:

$$f(v_i + v_o) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} K_{m,n} \cos(m\alpha) \cos(n\beta)$$

- Assume that the tank filters out all frequencies except the ones around ω_o. That means that only intermodulation terms that fall at ω_o are relevant:

$$\left| m\omega_i - n\omega_0 \right| = \omega_0$$

IL-Div [Fourier cont]

- If the injection signal is an *N*'th superharmonic, then only the intermodulation terms

$$n = Nm \pm 1$$

- possess a frequency equal to 1/*N* the incident frequency. This means that our summation can be written in terms of m as

$$u_{\omega_0}(t) = K_{0,1}\cos(\omega_0 t) + \frac{1}{2}\sum_{m=1}^{\infty} K_{m,Nm\pm 1}\cos(\omega_o t + m\phi)$$

- Using complex notation and applying the oscillation condition, the output signal can be written as

$$v_o = V_o e^{j\omega_0 t} = \frac{H_0 e^{j\omega_o t}}{1 + j2Q\frac{\Delta\omega}{\omega_r}}\left[K_{0,1} + \frac{1}{2}\sum_{m=1}^{\infty} K_{m,Nm\pm 1}e^{jm\phi}\right]$$

$$V_o\left(1 + j2Q\frac{\Delta\omega}{\omega_r}\right) = H_0\left[K_{0,1} + \frac{1}{2}\sum_{m=1}^{\infty} K_{m,Nm\pm 1}e^{jm\phi}\right]$$

Real Part:

$$V_o = H_0\left[K_{0,1} + \frac{1}{2}\sum_{m=1}^{\infty} K_{m,Nm\pm 1}\cos(m\phi)\right]$$

Imag Part:

$$2V_oQ\frac{\Delta\omega}{\omega_r} = \frac{H_0}{2}\sum_{m=1}^{\infty} K_{m,Nm\pm 1}\sin(m\phi)$$

IL-Div Locking Range

- The two equations can be solved for the unknown oscillation amplitude and phase for any incident amplitude and incident frequency, or any offset frequency:

$$\Delta\omega = (\omega_i/N) - \omega_r$$

- The second equation can be re-written as:

$$\Delta\omega = \Delta\omega_A \left[\frac{H_0}{2V_i} \sum_{m=1}^{\infty} K_{m,Nm\pm 1} \sin(m\phi) \right]$$

- where Adler's locking range has been identified:

$$\Delta\omega_A = \frac{\omega_r}{2Q} \frac{V_i}{V_o}$$

- Unlike static dividers, the locking range is limited.

IL Divide by 2 Circuit

- The equations can be solved analytically for a divide by 2 with cubic non-linearity

$$(\ ) = \ _0 + \ _1 \ + \ _2 \ ^2 + \ _3 \ ^3$$

$$\sin(\phi) = \frac{2Q}{H_0 a_2 V_i} \frac{\Delta\omega}{\omega_r}$$

$$|\sin(\phi)| < 1 \rightarrow \left|\frac{\Delta\omega}{\omega_r}\right| < \left|\frac{H_0 a_2 V_i}{2Q}\right|$$

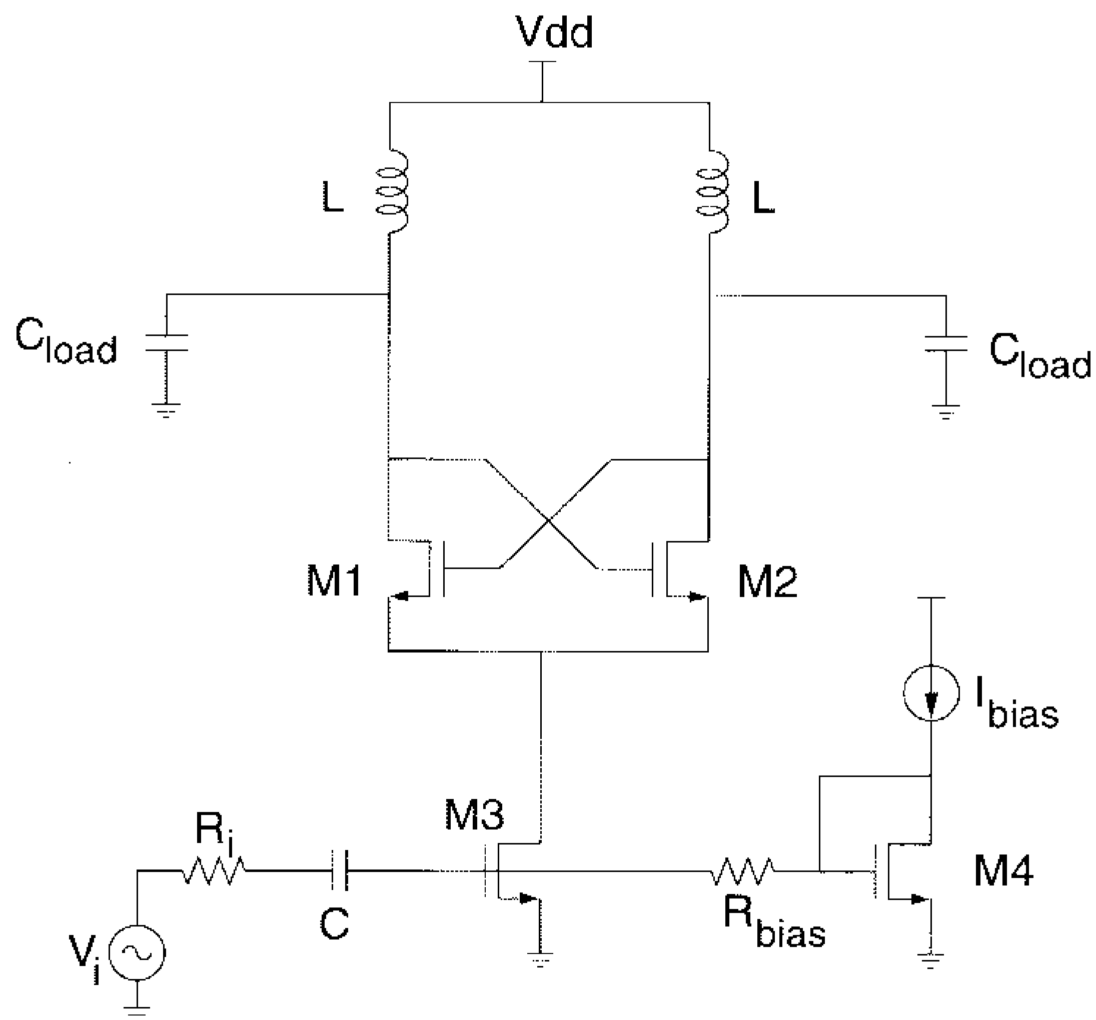
$$V_o = \sqrt{\frac{4}{3}\frac{1}{a_3 H_0}\left[1 - H_0\left(a_1 + \frac{3}{2}a_3 V_i^2 + a_2 V_i \cos(\phi)\right)\right]}$$

- Locking range is improved by using a large H0/Q or a larger injection amplitude. For an LC oscillator, this is equivalent to using a larger inductor:

$$\frac{H_0}{Q} = \omega L$$

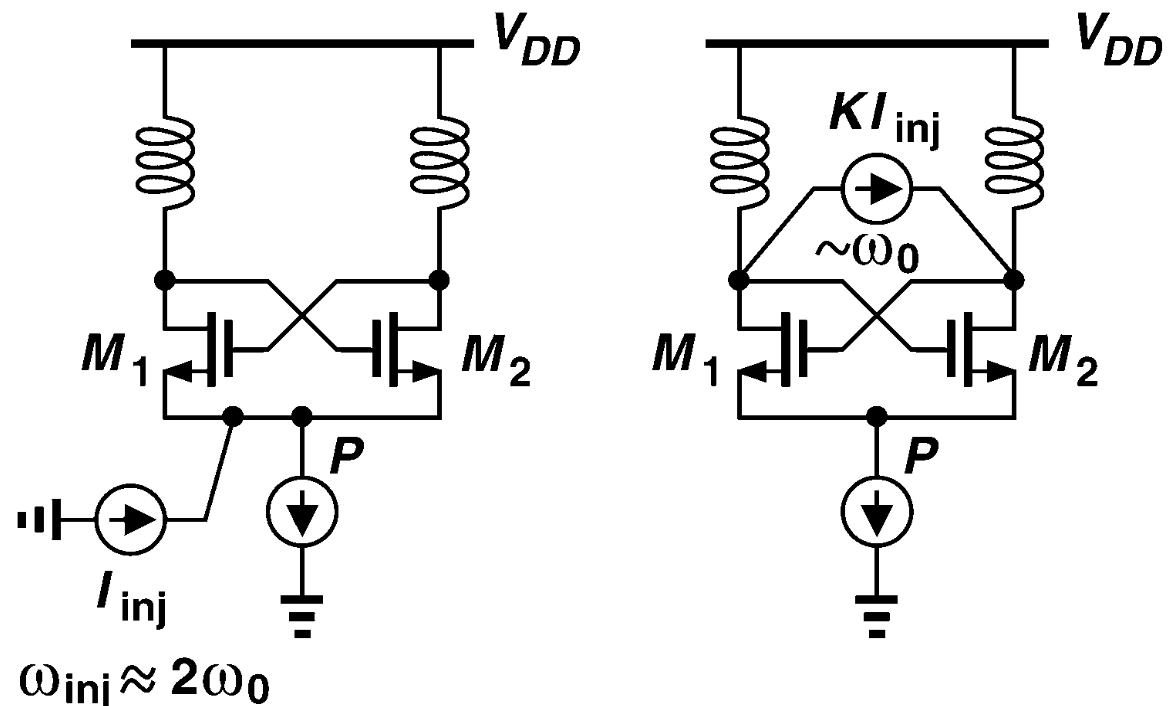
- A high impedance node is a convenient place to inject the signal to limit the injection power.

IL-Div Circuit Details



- The injection signal is applied as a current to the tail of a cross-coupled differential oscillator.
- The transistor currents of M1/M2 are a non-linear function of the output signal (feedback) and the injected current of M3.
- Interestingly, even in the absence of an injection signal, node M3 is moving at twice ω_o

IL-Div Intuitive Picture

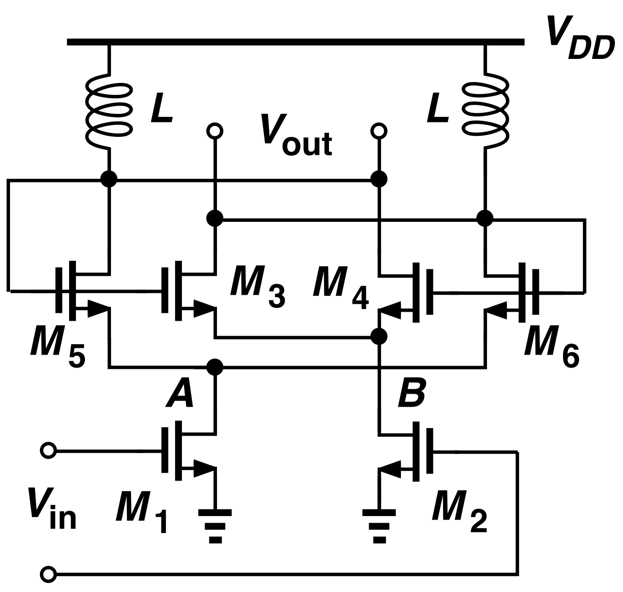


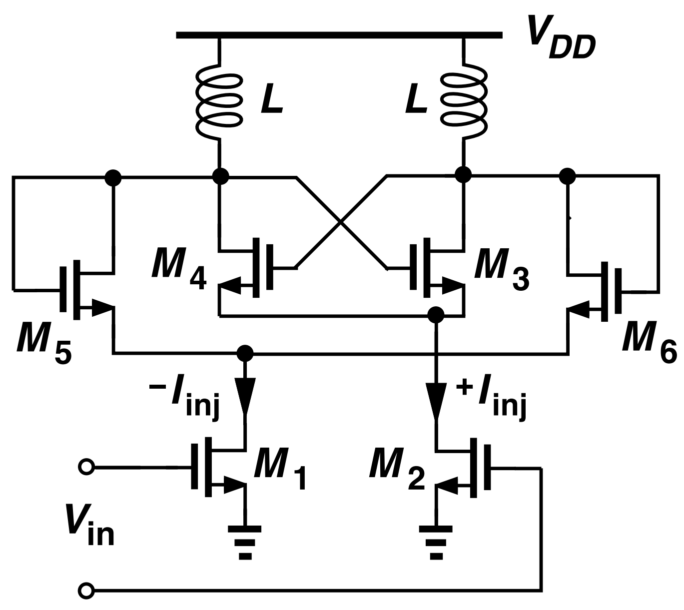
$$\omega_L \approx \frac{\omega_0}{2Q} \cdot \frac{2}{\pi} \cdot \frac{I_{\mathrm{inj}}}{I_{\mathrm{osc}}}.$$

$$\omega_{L,in} \approx \frac{\omega_0}{2Q} \cdot \frac{4}{\pi} \cdot \frac{I_{\mathrm{inj}}}{I_{\mathrm{osc}}}.$$

- Intuitively we can see that the injection at the tail of the MOS cross-coupled pair is mixed due to the switching action of M1/ M2.
- For a strong mixing signal, $2/\pi$ component of this current will flow into the tank at a frequency shift of harmonics of ω_o. In particular, a signal at 2 ω_o will be down-converted into ω_o, where the tank has high impedance. This signal therefore experiences a large loop gain and can lock the oscillator.

Miller Divider / Injection Locked Divider





- The same circuit topology can be seen to be an injection locked divider or a Miller divider.
- The devices M5/M6 are not needed in the normal injection locked divider, but here they act to lower the Q of the tank, increasing the lock range.
- A Miller divider can be designed so that it does not oscillate in the absence of an input signal.

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