EECS 142

Integrated Circuits for Communication

Crystal Oscillators (XTAL)

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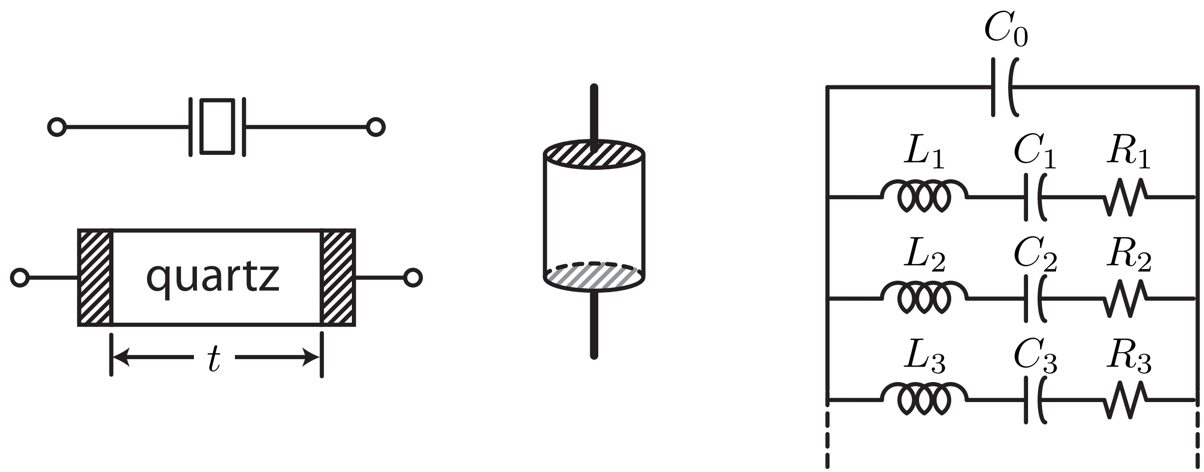
Crystal Resonator



- Quartz crystal is a piezoelectric material. An electric field causes a mechanical displacement and vice versa. Thus it is a electromechanical transducer.
- The equivalent circuit contains series LCR circuits that represent resonant modes of the XTAL. The capacitor C_0 is a physical capacitor that results from the parallel plate capacitance due to the leads.

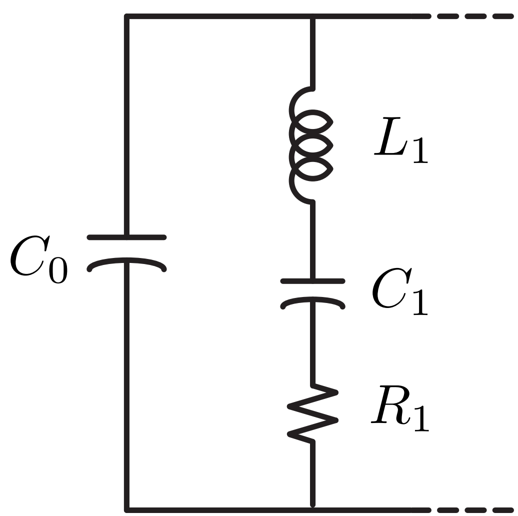
Fundamental Resonant Mode

- Acoustic waves through the crystal have phase velocity $v = 3 \times 10^3 \mathrm{m/s}$. For a thickness $t = 1\mathrm{mm}$, the delay time through the XTAL is given by $\tau = t/v = (10^{-3}\mathrm{m})/(3 \times 10^3 \mathrm{m/s}) = 1/3\mu\mathrm{s}$.
- This corresponds to a fundamental resonant frequency $f_0 = 1/\tau = v/t = 3\mathrm{MHz} = \frac{1}{2\pi\sqrt{L_1 C_1}}$.
- The quality factor is extremely high, with $Q \sim 3 \times 10^6$ (in vacuum) and about $Q = 1 \times 10^6$ (air). This is much higher than can be acheived with electrical circuit elements (inductors, capacitors, transmission lines, etc). This high Q factor leads to good frequency stability (low phase noise).

MEMS Resonators

- The highest frequency, though, is limited by the thickness of the material. For $t \approx 15\mu\mathrm{m}$, the frequency is about $200\mathrm{MHz}$. MEMS resonators have been demonstrated up to $\sim \mathrm{GHz}$ frequencies. MEMS resonators are an active research area.
- Integrated MEMS resonators are fabricated from polysilicon beams (forks), disks, and other mechanical structures. These resonators are electrostatically induced structures.
- We'll come back to MEMS resonators in the second part of the lecture

Example XTAL



- Some typical numbers for a fundamental mode resonator are $C_0 = 3\text{pF}$, $L_1 = 0.25\text{H}$, $C_1 = 40\text{fF}$, $R_1 = 50\Omega$, and $f_0 = 1.6\text{MHz}$. Note that the values of L_1 and C_1 are modeling parameters and not physical inductance/capacitance. The value of L is large in order to reflect the high quality factor.
- The quality factor is given by

$$Q = \frac{\omega L_1}{R_1} = 50 \times 10^3 = \frac{1}{\omega R_1 C_1}$$

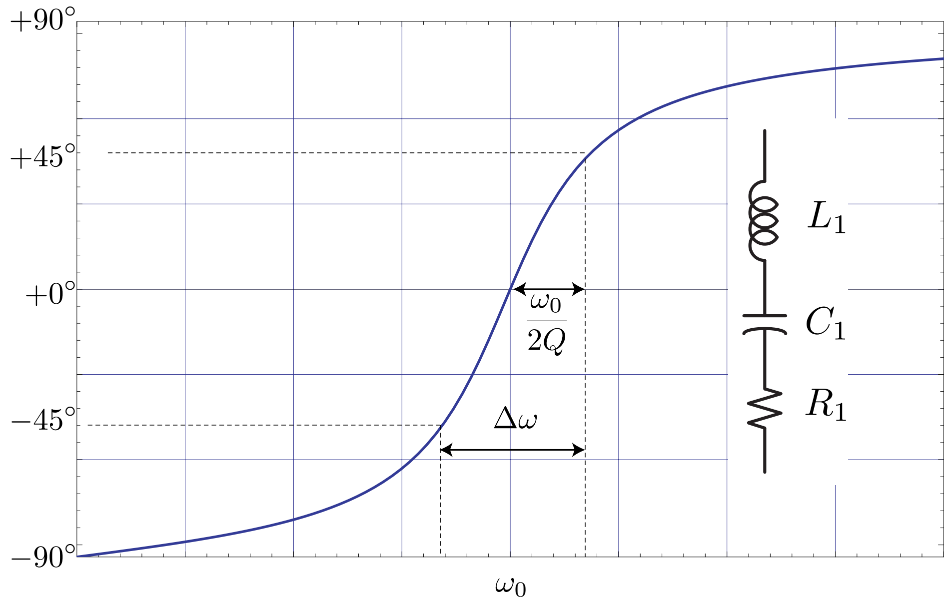
XTAL Resonance

- Recall that a series resonator has a phase shift from -90° to $+90^\circ$ as the impedance changes from capacitive to inductive form. The phase shift occurs rapidly for high Q structures.
- It's easy to show that the rate of change of phase is directly related to the Q of the resonator

$$Q = \frac{\omega_s}{2} \frac{d\phi}{d\omega}\bigg|_{\omega_0}$$

- For high Q structures, the phase shift is thus almost a "step" function unless we really zoom in to see the details.

XTAL Phase Shift

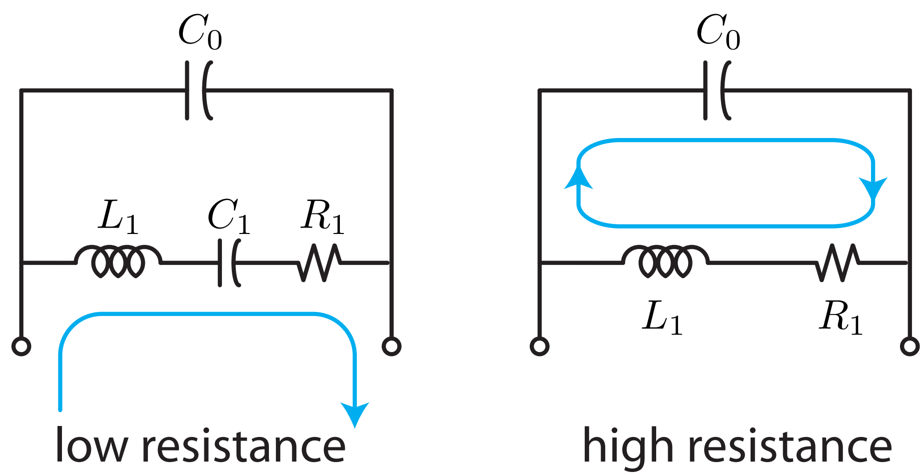


- In fact, it's easy to show that the $\pm 45^\circ$ points are only a distance of $\omega_s/(2Q)$ apart.

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$

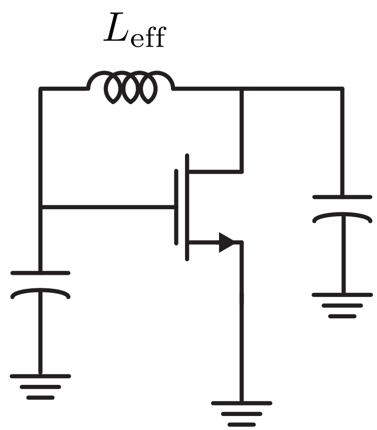
- For $Q = 50 \times 10^3$, this phase change requires an only 20ppm change in frequency.

Series and Parallel Mode



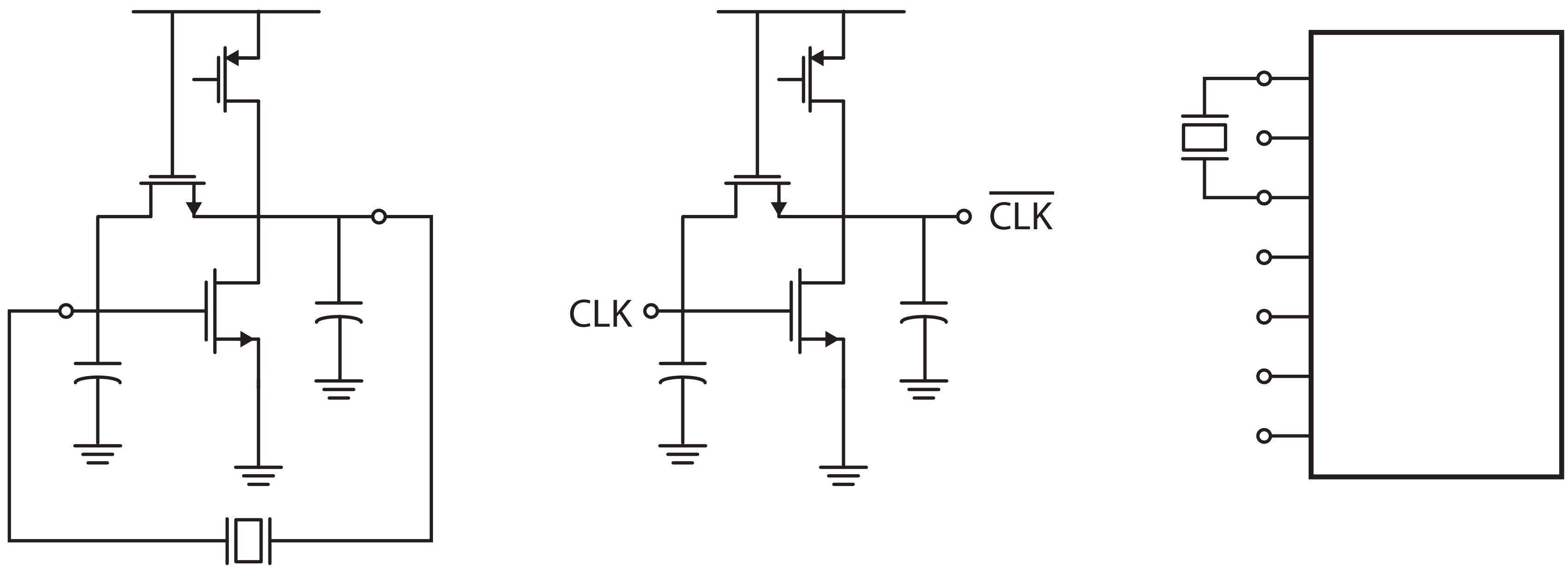
- Due to the external physical capacitor, there are two resonant modes between a series branch and the capacitor. In the series mode ω_s, the LCR is a low impedance ("short"). But beyond this frequency, the LCR is an equivalent inductor that resonates with the external capacitance to produce a parallel resonant circuit at frequency $\omega_p > \omega_s$.

Crystal Oscillator



- In practice, any oscillator topology can employ a crystal as an effective inductor (between ω_s and ω_p). The crystal can take on *any* appropriate value of L_{eff} to resonate with the external capacitance.
- Topoligies that minmize the tank loading are desirable in order to minize the XTAL de-Qing. The Pierce resonator is very popular for this reason.

Clock Application

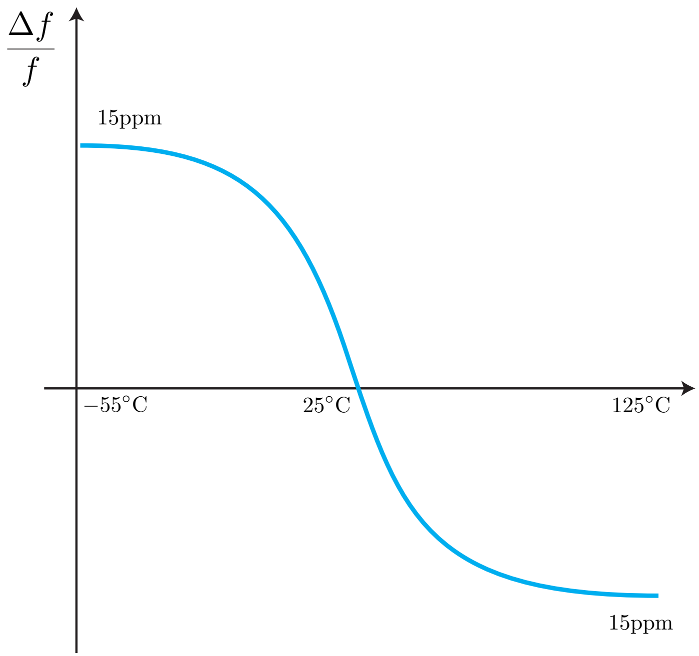


- Note that if the XTAL is removed from this circuit, the amplifier acts like a clock driver. This allows the flexbility of employing an external clock or providing an oscillator at the pins of the chip.

XTAL Tempco

- The thickness has tempco $t \sim 14\text{ppm}/^\circ\text{C}$ leading to a variation in frequency with temperature. If we cut the XTAL in certain orientations ("AT-cut") so that the tempco of velocity cancels tempco of t, the overall tempco is minimized and a frequency stability as good as $f_0 \sim 0.6\text{ppm}/^\circ\text{C}$ is possible.
- Note that $1\text{sec}/\text{mo} = 0.4\text{ppm}$! Or this corresponds to only 0.4Hz in 1MHz.
- This change in thickness for 0.4ppm is only $\delta t = 0.4 \times 10^{-6} \times t_0 = 0.4 \times 10^{-6} \times 10^{-3}\text{m} = 4 \times 10^{-10}$. That's about 2 atoms!
- The smallest form factors available today's AT-cut crystals are 2×1.6 mm^2 in the frequency range of 24-54 MHz are available.

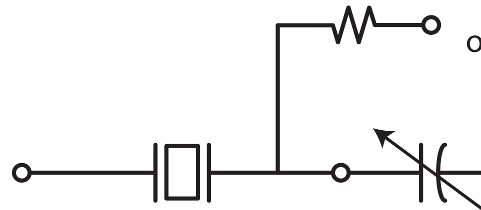
OCXO



- The typical temperature variation of the XTAL is shown. The variation is minimized at room temperature by design but can be as large as 15ppm at the extreme ranges.

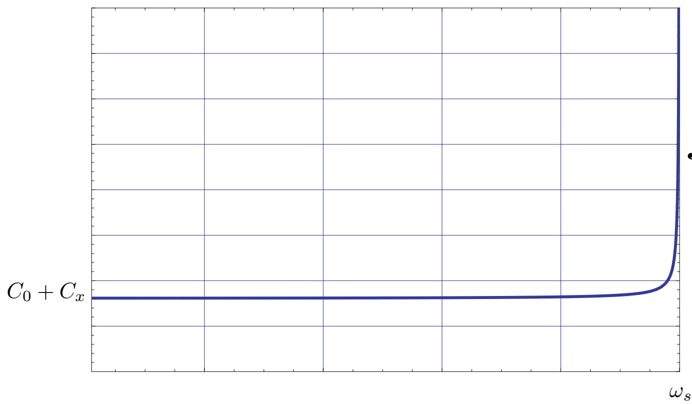
- To minimize the temperature variation, the XTAL can be placed in an oven to form an *Oven Compensated XTAL Oscillator*, or OCXO. This requires about a cubic inch of volume but can result in extremely stable oscillator. $\text{OCXO} \sim 0.01\text{ppm}/^\circ\text{C}$.

TCXO



- In many applications an oven is not practical. A *Temperature Compenstated XTAL Oscillator*, or TCXO, uses external capacitors to “pull” or “push” the resonant frequency. The external capacitors can be made with a varactor.
- This means that a control circuit must estimate the operating temperature, and then use a pre-programmed table to generate the right voltage to compensate for the XTAL shift.
- This scheme can acheive as low as $\mathrm{TCXO} \sim 0.05\mathrm{ppm}/^\circ\mathrm{C}$.
- Many inexpensive parts use a DCXO, or a digitally-compensated crystal oscillator, to eliminate the TCXO. Often a simple calibration procedure is used to set the XTAL frequency to within the desired range and a simple look-up table is used to adjust it.

XTAL Below ω_s



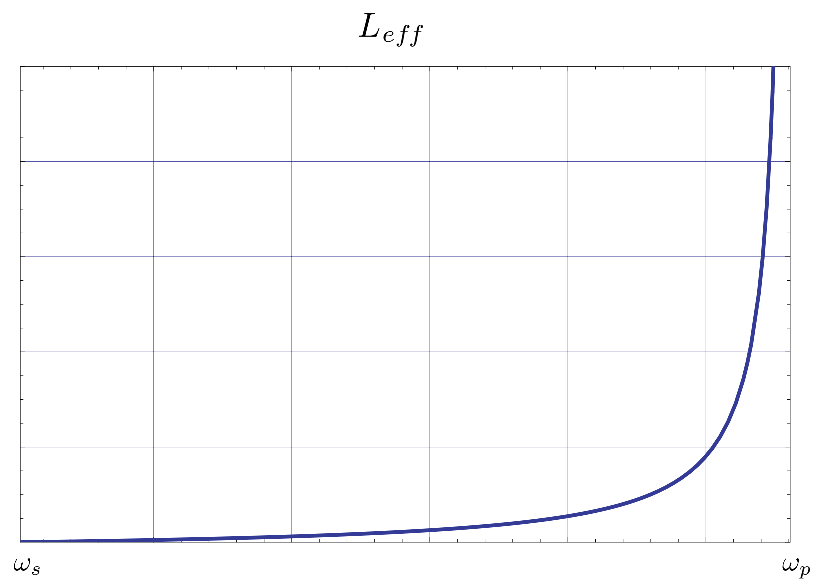
$$jX_c = \frac{1}{j\omega C_{eff}} = \frac{1}{\omega C_0} || \left(\frac{1}{j\omega C_x} + j\omega L_x \right)$$

$$= \frac{1}{j\omega C_0} || \frac{1}{j\omega C_x} \left(1 - \omega^2 L_x C_x\right)$$

- Below series resonance, the equivalent circuit for the XTAL is a capacitor is easily derived.
- The effective capacitance is given by

$$C_{eff} = C_0 + \frac{C_x}{1 - \left(\frac{\omega}{\omega_s}\right)^2}$$

XTAL Inductive Region



- Past series resonance, the XTAL reactance is inductive

$$jX_c = j\omega L_{eff} = \frac{1}{j\omega C_0} || j\omega L_x \left(1 - \left(\frac{\omega_s}{\omega}\right)^2\right)$$

- The XTAL displays L_{eff} from $0 \rightarrow \infty \mathrm{H}$ in the range from $\omega_s \longrightarrow \omega_p$.
- Thus for *any* C, the XTAL will resonate somewhere in this range.

Inductive Region Frequency Range

- We can solve for the frequency range of (ω_s, ω_p) using the following equation

$$j\omega_p L_{eff} = \frac{1}{j\omega_p C_0} || j\omega_p L_x \left(1 - \left(\frac{\omega_s}{\omega_p}\right)^2\right)$$

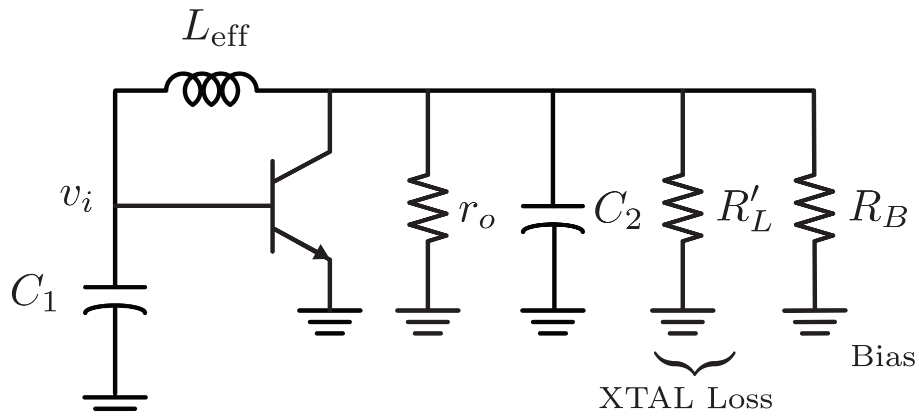
$$\vdots$$

$$\frac{\omega_p}{\omega_s} = \sqrt{1 + \frac{C_x}{C_0}}$$

- Example: $C_x = 0.04\text{pF}$ and $C_0 = 4\text{pF}$ Since $C_0 \gg C_x$, the frequency range is very tight

$$\frac{\omega_p}{\omega_s} = 1.005$$

XTAL Losses



- Consider now the series losses in the XTAL. Let $X_1 = -1/(\omega C_1)$ and $X_2 = -1/(\omega C_2)$, and $jX_c = j\omega L_{eff}$. Then the impedance Z_L' is given by

$$Z_L' = \frac{jX_1(R_x + jX_c + jX_2)}{R_x + \underbrace{(jX_c + jX_1 + jX_2)}_{\equiv 0 \text{ at resonance}}}$$

Reflected Losses

- It follows therefore that $jX_c + jX_2 = -jX_1$ at resonance and so

$$Z_L' = \frac{jX_1(R_x - jX_1)}{R_x} = (X_1 + jR_x)\frac{X_1}{R_x}$$

- Since the Q is extremely high, it's reasonable to assume that $X_c \gg R_x$ and thus $X_1 + X_2 \gg R_x$, and if these reactances are on the same order of magnitude, then $X_1 \gg R_x$. Then

$$Z_L' \approx \frac{X_1^2}{R_x}$$

- This is the XTAL loss reflected to the output of the oscillator.

Losses at Overtones

- Since X_1 gets smaller for higher ω, the shunt loss reflected to the output from the overtones gets smaller (more loading).
- The loop gain is therefore lower at the overtones compared to the fundamental in a Pierce oscillator.
- For a good design, we ensure that $A_\ell < 1$ for all overtones so that only the dominate mode oscillates.

XTAL Oscillator Design

- The design of a XTAL oscillator is very similar to a normal oscillator. Use the XTAL instead of an inductor and reflect all losses to the output.

$$A_\ell = g_m R_L \frac{C_1}{C_2}$$

$$R_L = R_L' || R_B || r_o || \cdots$$

- For the steady-state, simply use the fact that $G_m R_L \frac{C_1}{C_2} = 1$, or $G_m / g_m = 1/A_\ell$.

XTAL Oscillator Simulation

- For a second order system, the poles are placed on the circle of radius ω_0. Since the envelope of a small perturbation grows like

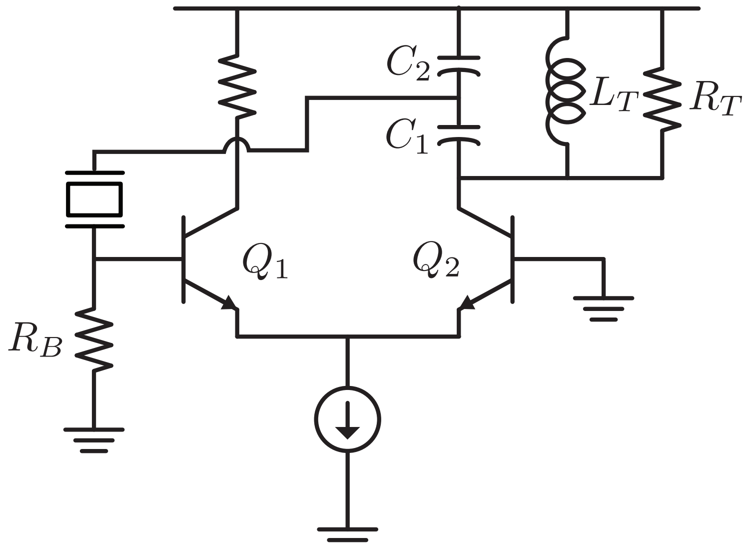
$$v_0(t) = Ke^{\sigma_1 t}\cos\omega_0 t$$

- where $\sigma_1 = 1/\tau$ and $\tau = \frac{Q}{\omega_0}\frac{2}{A_\ell - 1}$.
- For example if $A_\ell = 3$, $\tau = \frac{Q}{\omega_0}$. That means that if $Q \sim 10^6$, about a million cycles of simulation are necessary for the amplitude of oscillation to grow by a factor of $e \approx 2.71$!

PSS/HB versus TRAN

- Since this can result in a very time consuming transient (TRAN) simulation in SPICE, you can artifically de-Q the tank to a value of $Q \sim 10 - 100$. Use the same value of R_x but adjust the values of C_x and L_x to give the right ω_0 but low Q.
- Alternatively, if PSS or harmonic balance (HB) are employed, the steady-state solution is found directly avoiding the start-up transient.
- Transient assisted HB and other techniques are described in the ADS documentation.

Series Resonance XTAL



- Note that the LCR tank is a low $Q(3-20)$ tuned to the approximate desired fundamental frequency of the XTAL (or overtone).

- The actual frequency selectivity comes from the XTAL, not the LCR circuit. The LCR loaded Q at resonance is given by the reflected losses at the tank

$$R_T' = R_T || n^2 (R_x + R_B') \qquad R_B' = R_B || R_{i|DP}$$

Series XTAL Loop Gain

- At resonance, the loop gain is given by

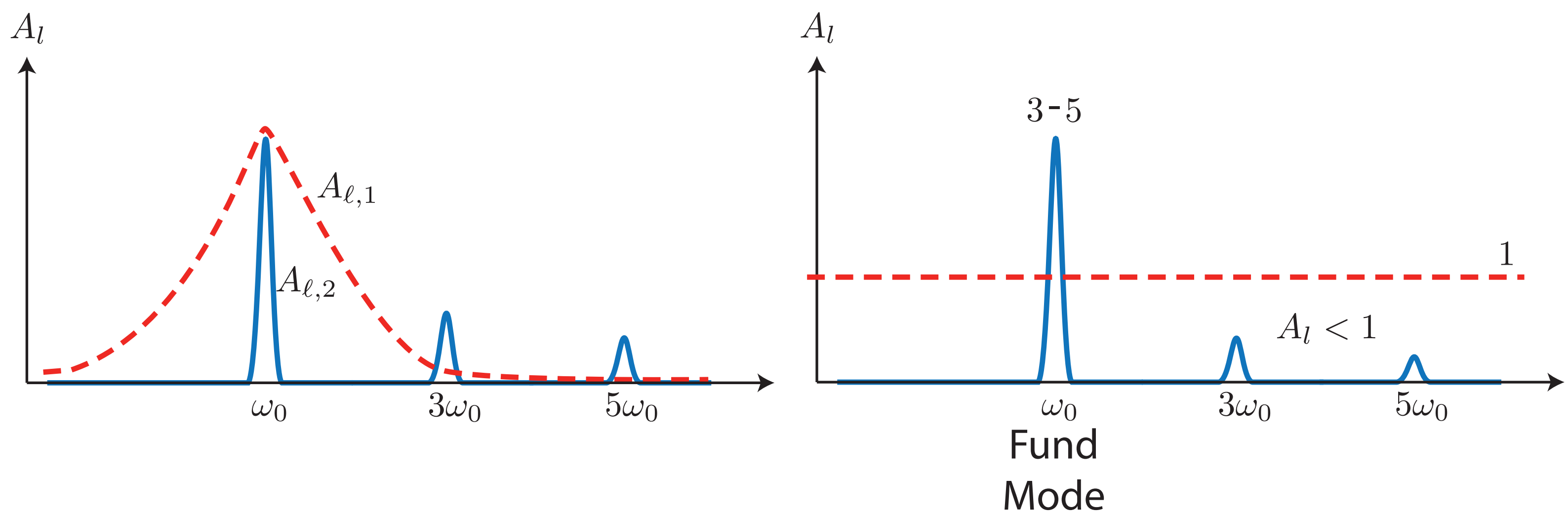
$$A_\ell = G_m R_T' \frac{C_1}{C_1 + C_2} \frac{R_B'}{R_B' + R_x}$$

- The last term is the resistive divider at the base of Q1 formed by the XTAL and the biasing resistor.
- In general, the loop gain is given by

$$A_\ell = \underbrace{G_m Z_T(j\omega) \frac{C_1}{C_1 + C_2}}_{A_{\ell,1}} \underbrace{\frac{R_B'}{R_B' + Z_x(j\omega)}}_{A_{\ell,2}}$$

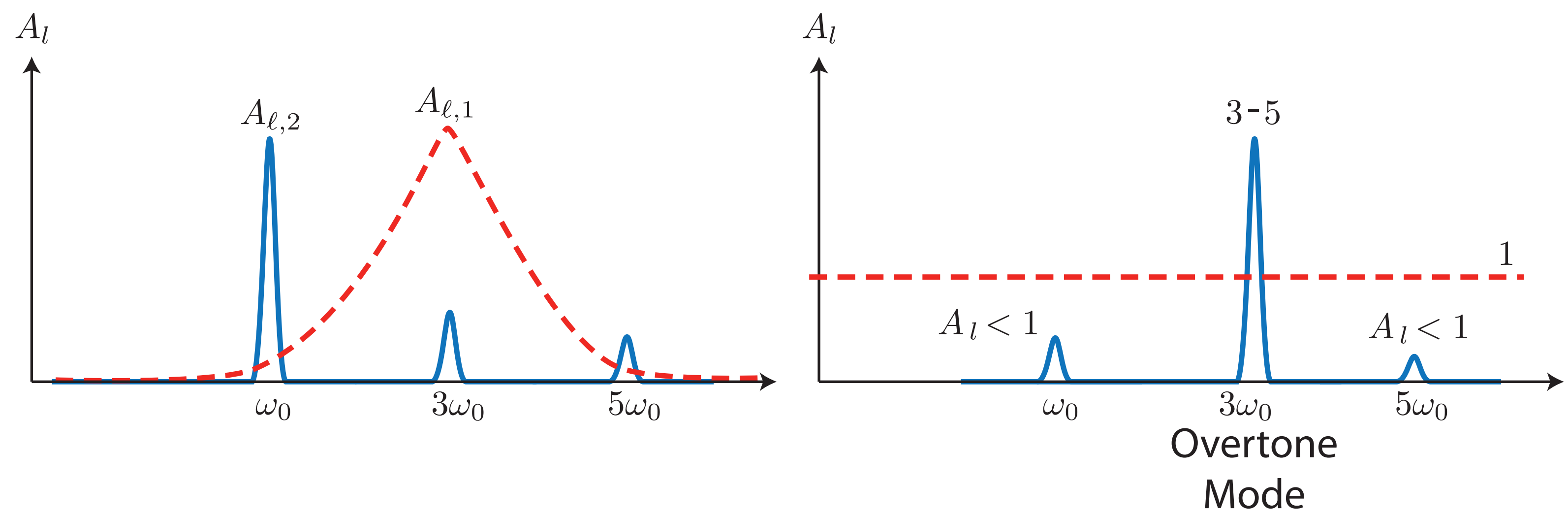
- The first term $A_{\ell,1}$ is not very frequency selective due to the low Q tank. But $A_{\ell,2}$ changes rapidly with frequency.

Series XTAL Fundametal Mode



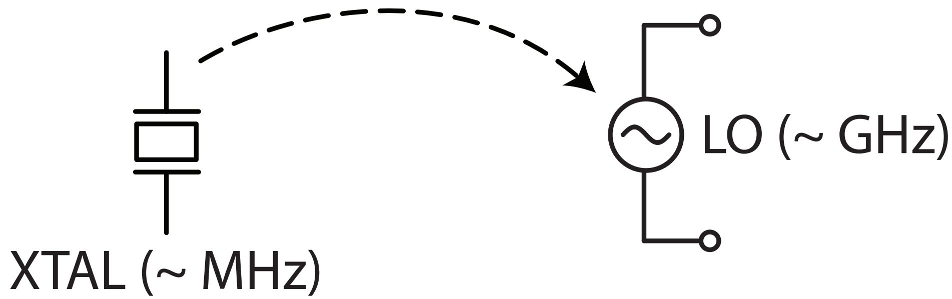
- In this case the low Q tank selects the fundamental mode and the loop gain at all overtones is less than unity.

Series XTAL Overtone Mode



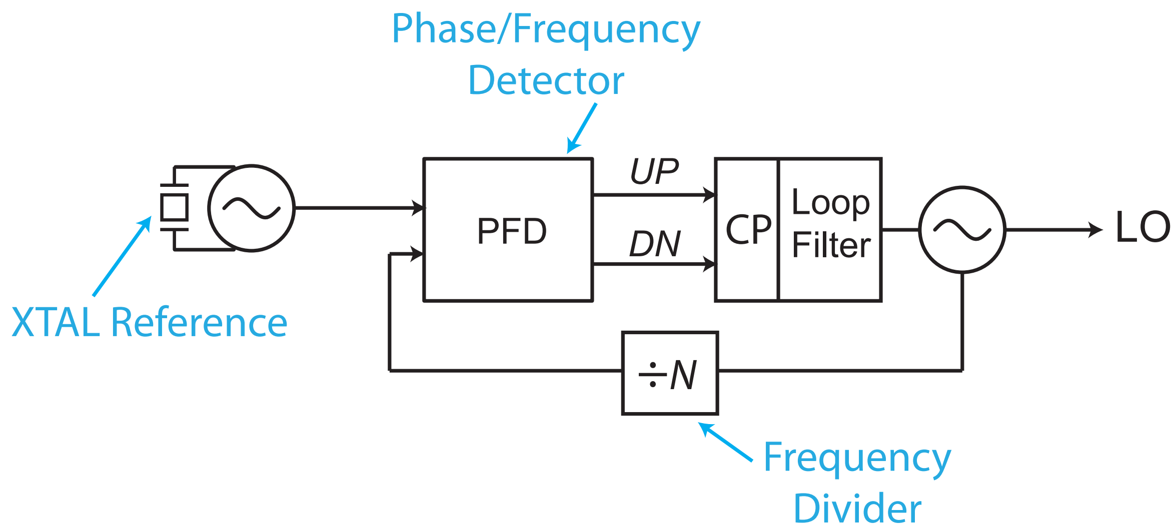
- In this case the low Q tank selects a $3\omega_0$ overtone mode and the loop gain at all other overtones is less than unity. The loop gain at the fundamental is likewise less than unity.

Frequency Synthesis



- For communication systems we need precise frequency references, stable over temperature and process, with low phase noise. We also need to generate different frequencies "quickly" to tune to different channels.
- XTAL's are excellent references but they are at lower frequencies (say below $200\mathrm{MHz}$) and fixed in frequency. How do we synthesize an RF and variable version of the XTAL?

PLL Frequency Synthesis



- This is a “phase locked loop” frequency synthesizer. The stable XTAL is used as a reference. The output of a VCO is phase locked ot this stable reference by dividing the VCO frequency to the same frequency as the reference.
- The phase detector detects the phase difference and generates an error signal. The loop filter thus will force phase equality if the feedback loop is stable.

MEMS Reference Oscillators

EECS 242: Lecture 25

Prof. Ali M. Niknejad

Why replace XTAL Resonators?

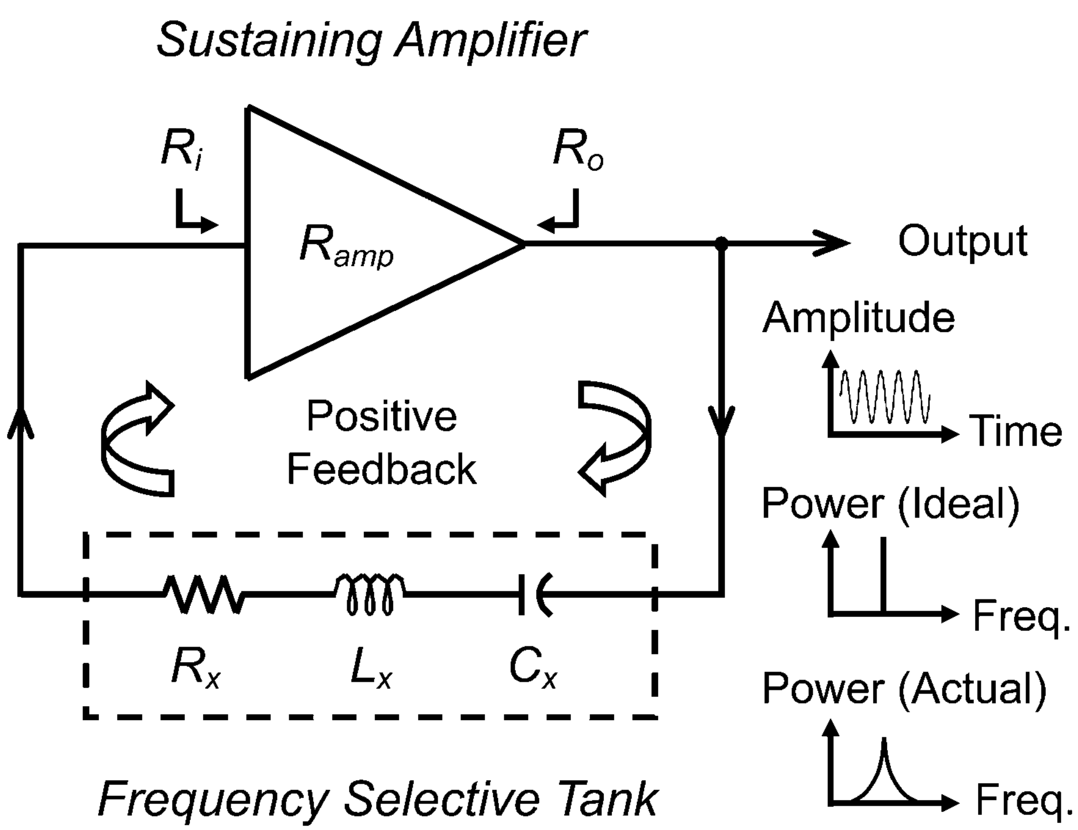
- XTAL resonators have excellent performance in terms of quality factor (Q ~ 100,000), temperature stability (< 1 ppm/C), and good power handling capability (more on this later)
- The only downside is that these devices are bulky and thick, and many emerging applications require much smaller form factors, especially in thickness (flexible electronics is a good example)
- MEMS resonators have also demonstrated high Q and Si integration (very small size) ... are they the solution we seek?
- Wireless communication specs are very difficult:
  - GSM requires -130 dBc/Hz at 1 kHz from a 13 MHz oscillator
  - -150 dBc/Hz for far away offsets

Business Opportunity

- XTAL oscillators is a $4B market. Even capturing a small chunk of this pie is a lot of money.
- This has propelled many start-ups into this arena (SiTime, SiClocks, Discera) as well as new approaches to the problem (compensated *LC* oscillators) by companies such as Mobius and Silicon Labs
- Another observation is that many products in the market are programmable oscillators/timing chips that include the PLL in the package.
- As we shall see, a MEMS resonator does not make sense in a stand-alone application (temp stability), but if an all Si MEMS based PLL chip can be realized, it can compete in this segment of the market

Series Resonant Oscillator

- The motional resistance of MEMS resonators is quite large (typically koms compared to ohms for XTAL) and depends on the fourth power of gap spacing
- This limits the power handling capability
- Also, in order not to de-Q the tank, an amplifier with low input/output impedance is required. A trans-resistance amplifier is often used



$$R_{amp} \geq R_x + R_i + R_o = R_{tot}$$

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Zero'th Order Leeson Model

$$L\left\{f_m\right\} = \frac{2kT(1+F_{Ramp})}{P_o} \cdot \left(\frac{R_{tot}}{R_x}\right) \cdot \left[1 + \left(\frac{f_0}{2Q_l \cdot f_m}\right)^2\right]$$

$$Q_l = \frac{R_x}{R_x + R_i + R_o} Q = \frac{R_x}{R_{tot}} Q$$

- Using a simple Leeson model, the above expression for phase noise is easily derived.
- The insight is that while MEMS resonators have excellent Q's, their power handling capability will ultimately limit the performance.
- Typically MEMS resonators amp limit based on the non-linearity of the resonator rather than the electronic non-linearities, limiting the amplitude of the oscillator

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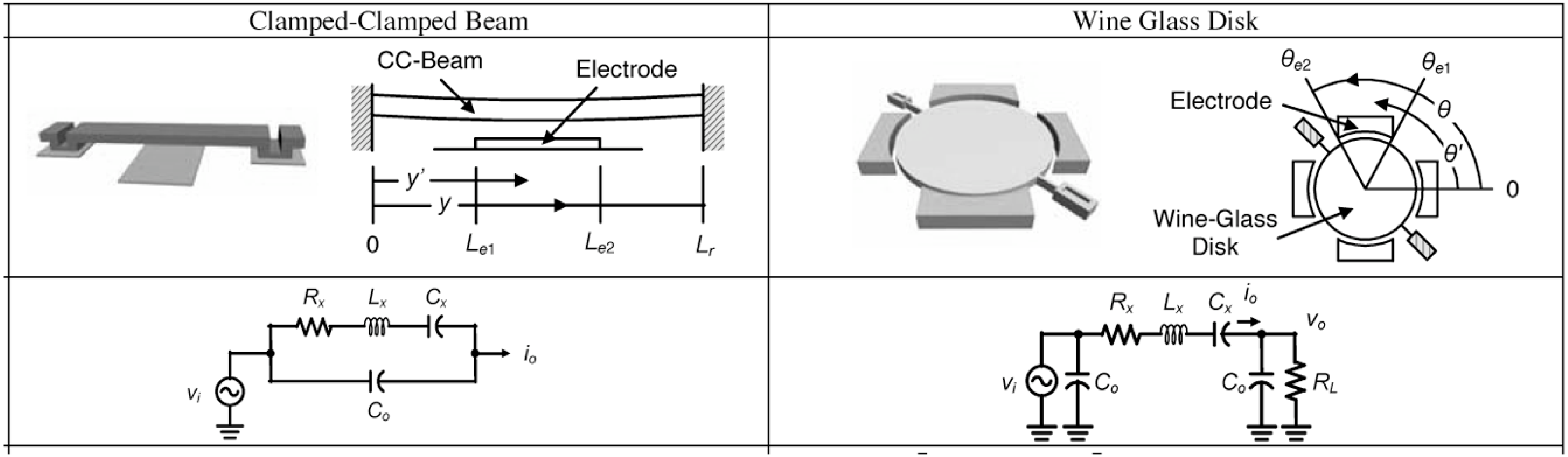
MEMS Resonator Designs



- Clampled-clamped beam and wine disk resonator are very populator. Equivalent circuits calculated from electromechanical properties.
- Structures can be fabricated from polysilicon (typical dimensions are small ~ 10 um)
- Electrostatic transduction is used (which requires large voltages > 10V).

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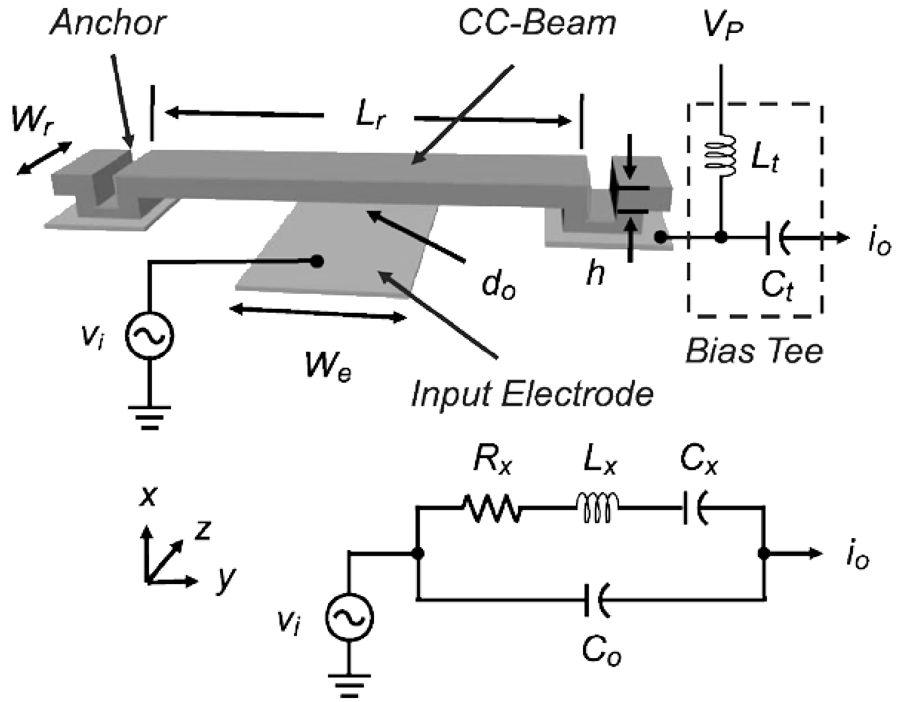
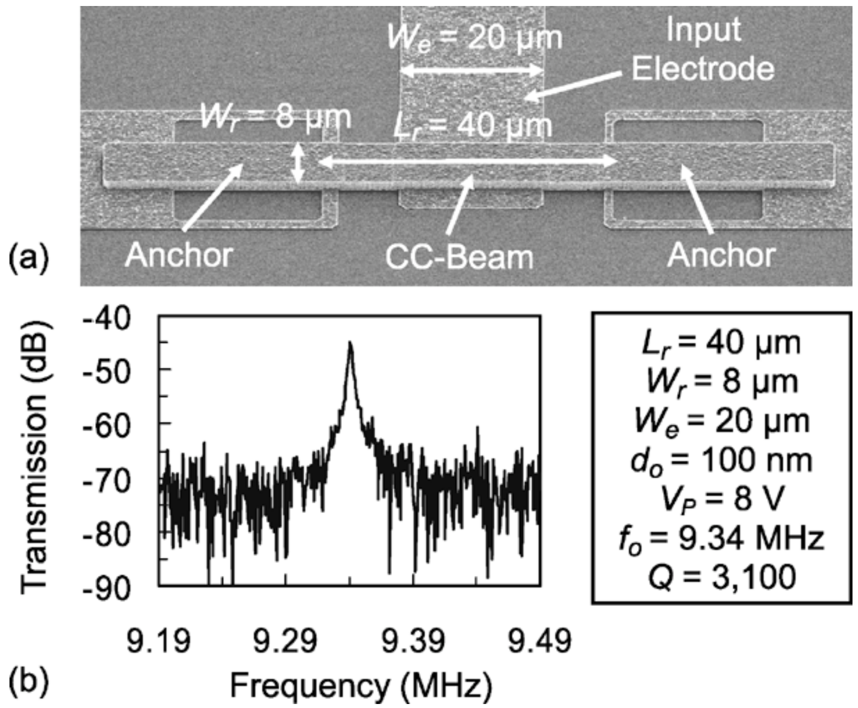
CC-Beam Resonator





- This example uses an 8-μm wide beamwidth and a 20-μm wide electrode.
- Measurements are performed in vacuum.
- Q ~ 3000 for a frequency of 10 MHz

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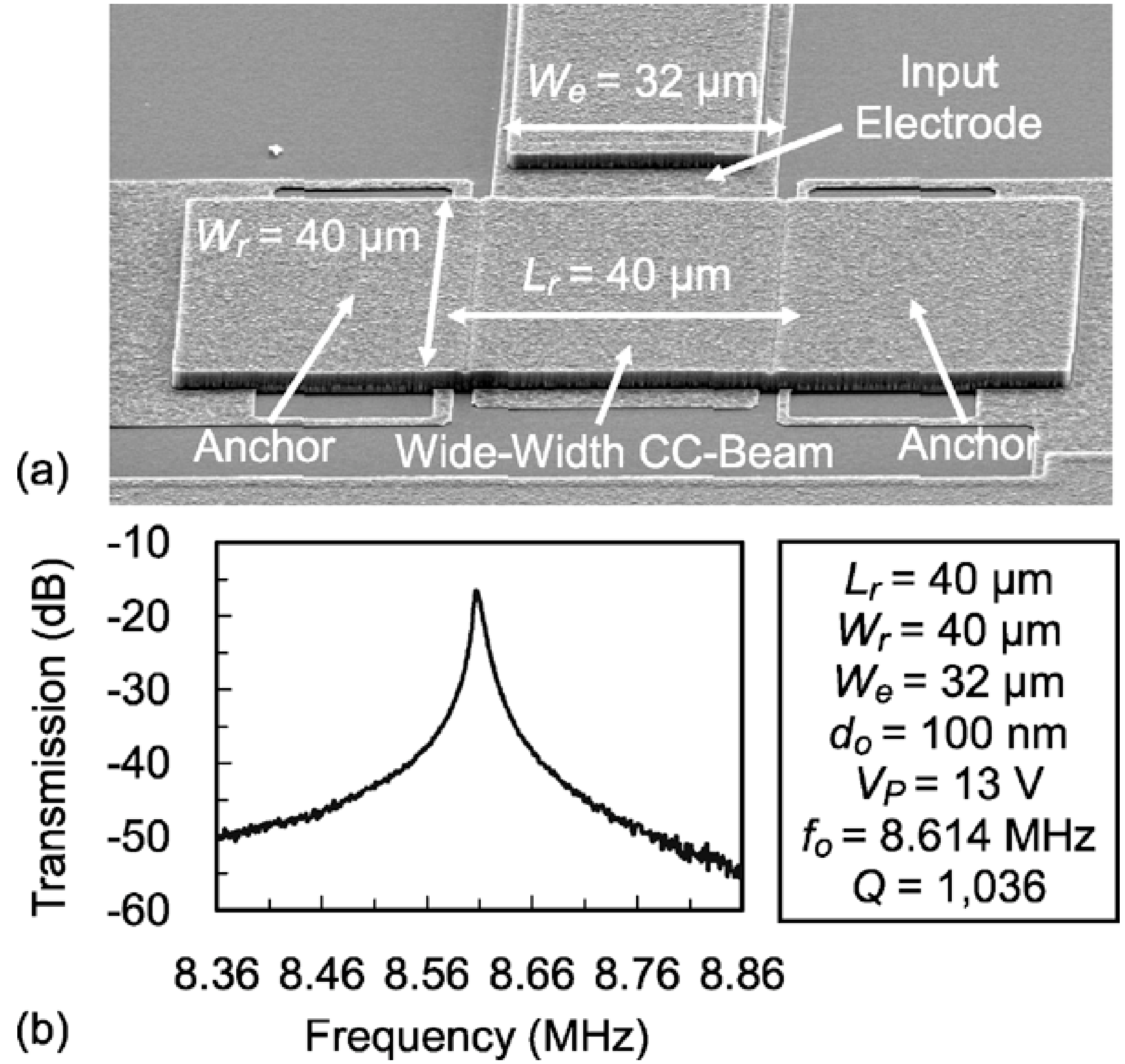
CC-Beam with Better Power Handling



- To increase power handling of the resonator, a wider beam width is used [~10X in theory].
- The motional resistance is reduced to 340 ohms (Vp = 13V)

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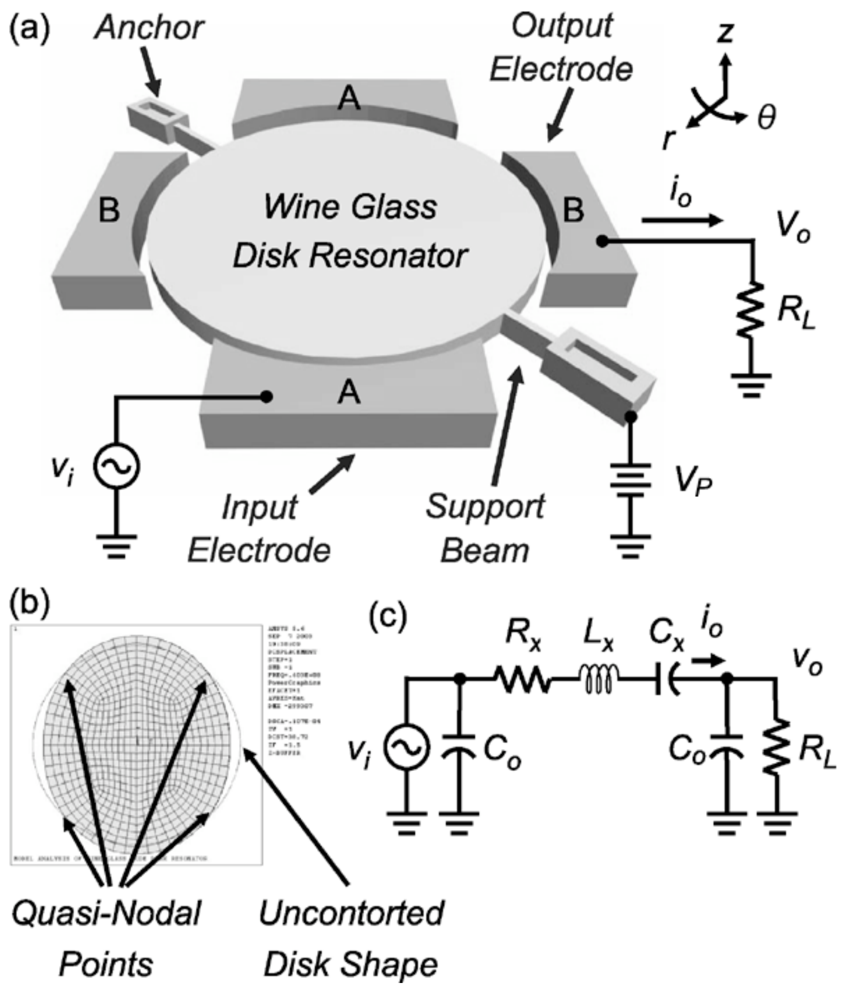
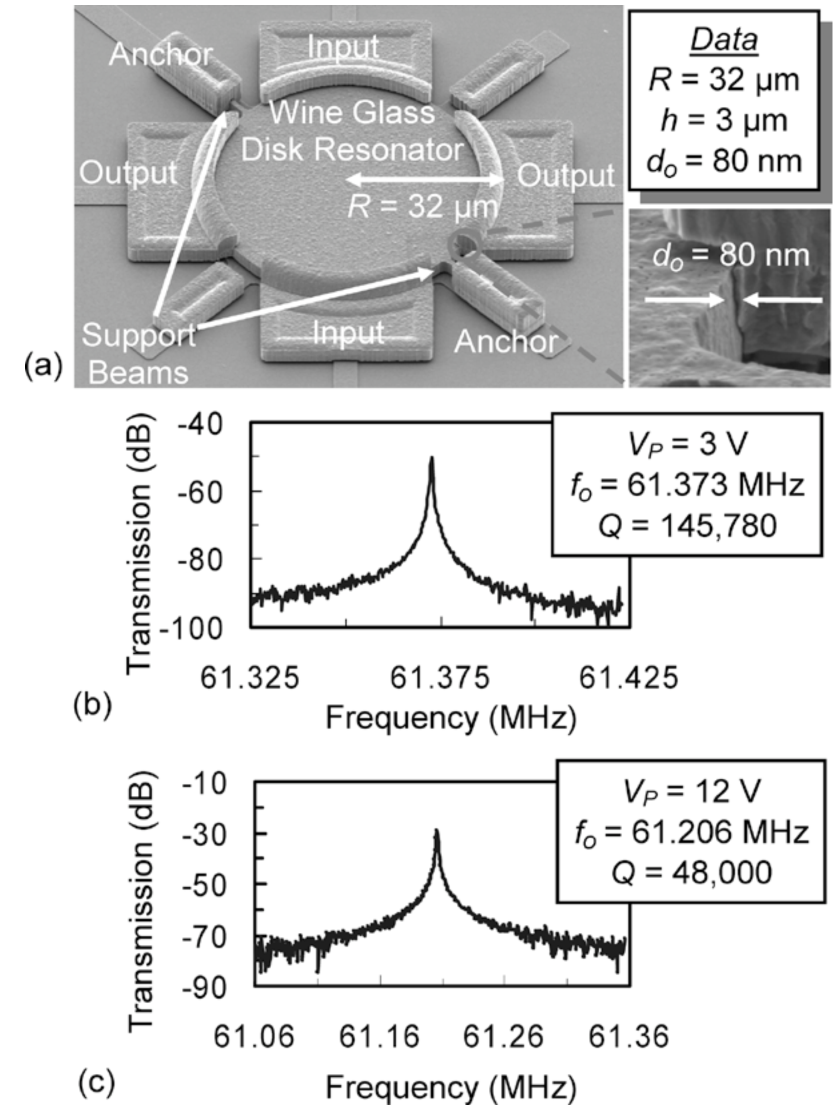
Disk Wineglass Resonator





- Intrinsically better power handling capability from a wine glass resonator.
- The input/output ports are isolated (actuation versus sensing).

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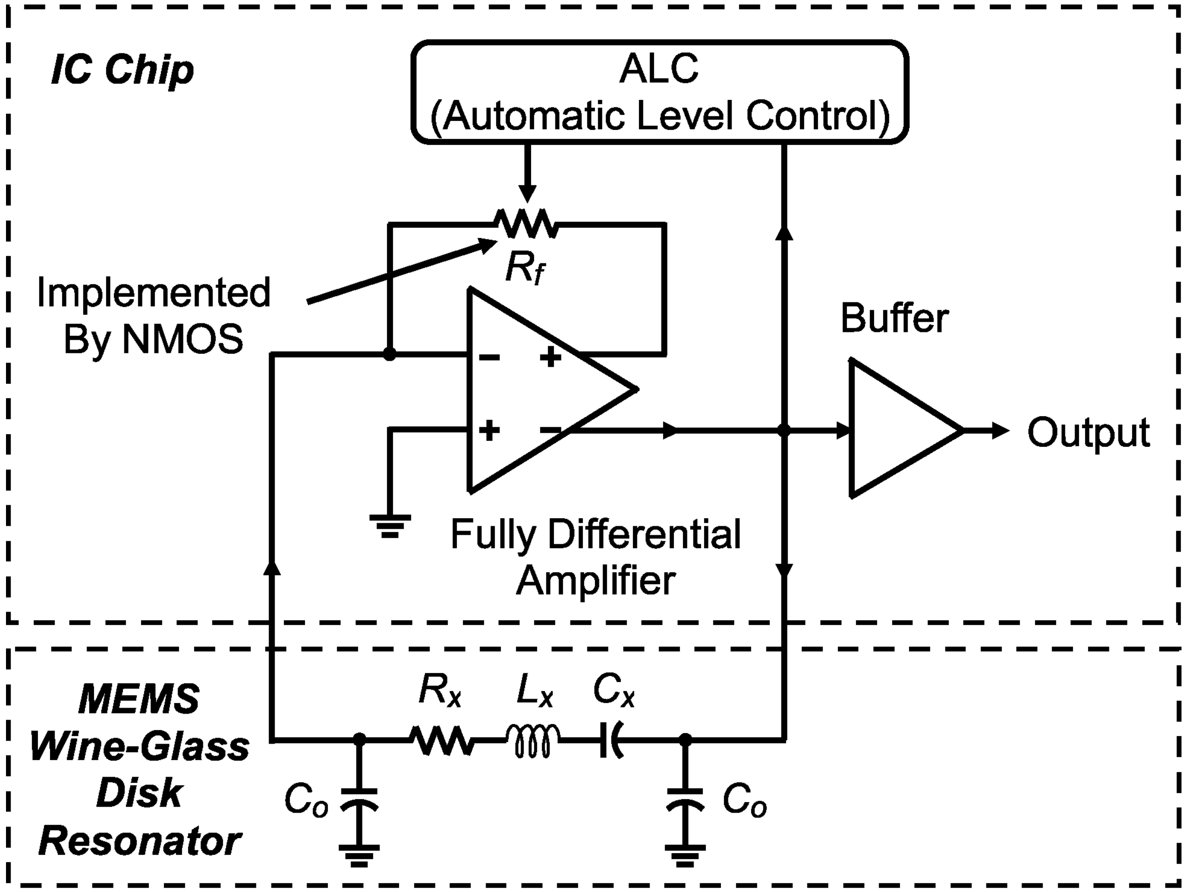
Sustaining Amplifier Design



- Use feedback amplifier to create positive feedback trans-resistance
- Automatic gain control is used so that the oscillation self-limits through the electronic non-linearity. This reduces the oscillator amplitude but also helps to reduce 1/f noise up-conversion

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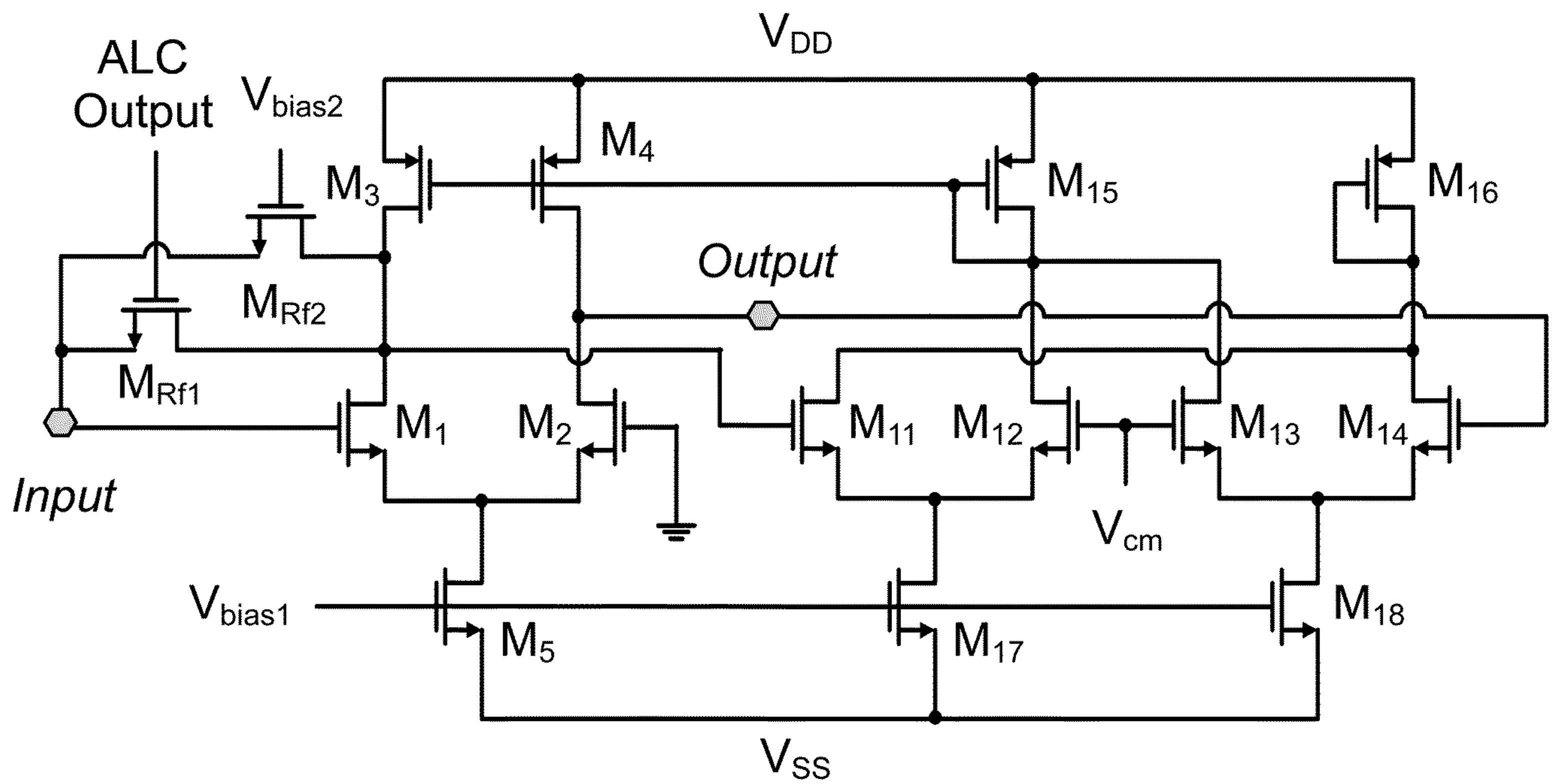
Amplifier Details



- Single-stage amplifier is used to maximize bandwidth. Recall that any phase shift through the amplifier causes the oscillation frequency to shift (and phase noise to degrade)
- Common-mode feedback used to set output voltage. Feedback resistance and Amplitude Level Control (ALC) implemented with MOS resistors

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Design Equations

$$R_{\mathrm{amp}} = \frac{\frac{1}{2}g_{m1}(R_f//r_{o1}//r_{o3})R_f}{1+\frac{1}{2}g_{m1}(R_f//r_{o1}//r_{o3})} \cong \frac{g_{m1}R_f^2}{2+g_{m1}R_f} \cong R_f$$

$$R_i = \frac{R_f}{1+\frac{1}{2}g_{m1}(R_f//r_{o1}//r_{o3})} \cong \frac{2R_f}{2+g_{m1}R_f} \cong \frac{2}{g_{m1}}$$

$$R_o = \frac{R_f//r_{o1}//r_{o3}}{1+\frac{1}{2}g_{m1}(R_f//r_{o1}//r_{o3})} \cong \frac{2R_f}{2+g_{m1}R_f} \cong \frac{2}{g_{m1}}$$

$$\frac{\overline{v_{ia}^2}}{\Delta f} = 4kT \cdot \gamma \cdot \frac{2}{g_{m1}} \cdot \left(1 + \frac{g_{m3}}{g_{m1}}\right)$$

$$R_{\mathrm{amp}}(s) = \frac{r_m}{1 + r_m \cdot \frac{1}{R_f}} = \frac{R_f \omega_i \omega_b a_v}{s^2 + s(\omega_i + \omega_b) + \omega_i \omega_b (1 + a_v)}$$

- These equations are used to trade-off between power and noise in the oscillator. The device size cannot be too large since the bandwidth needs to be about 10X the oscillation frequency.

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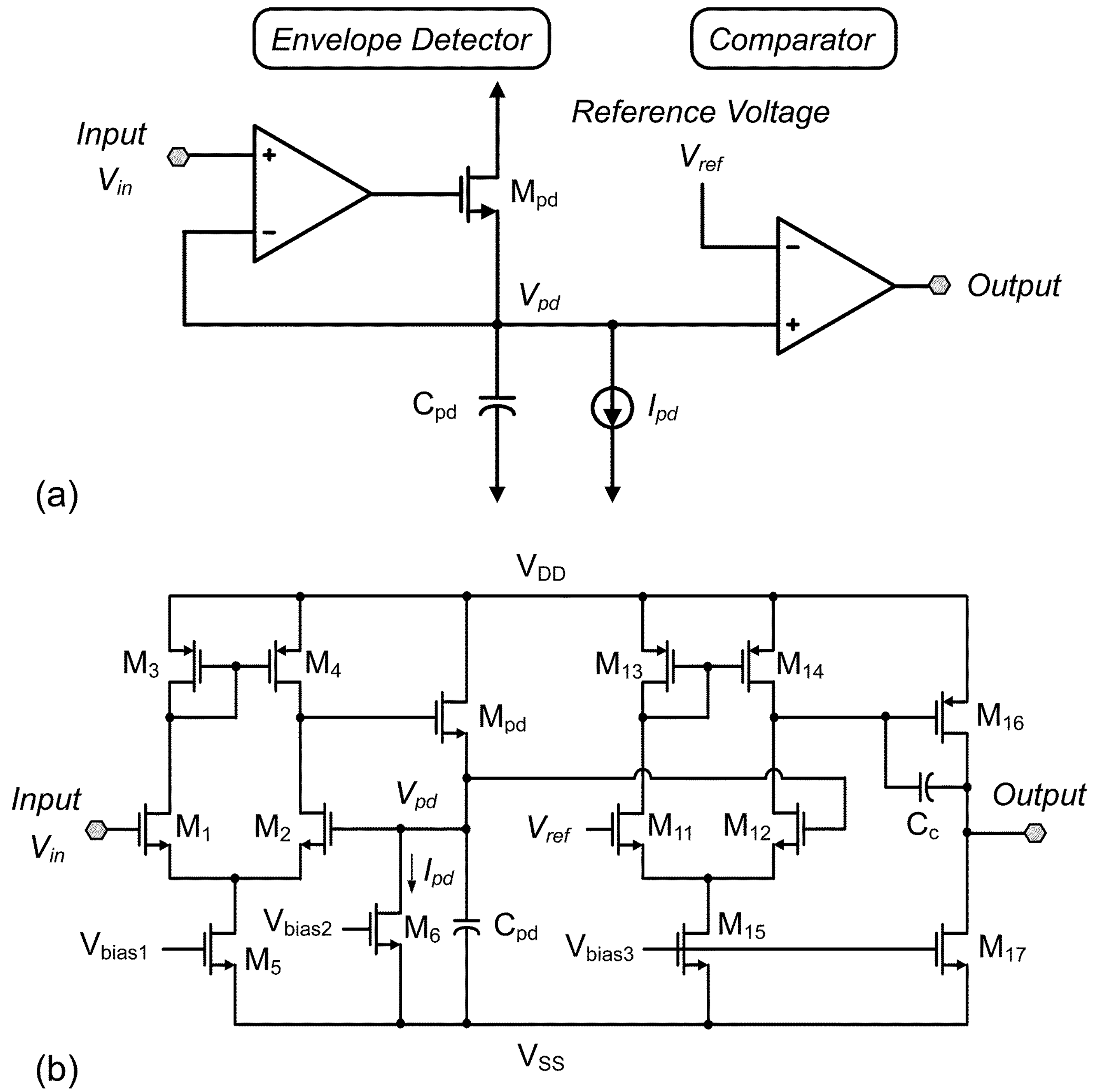
Amplitude Control Loop

- Precision peak-detector used to sense oscillation amplitude. This is done by putting a MOS diode in the feedback path of an inverting op-amp



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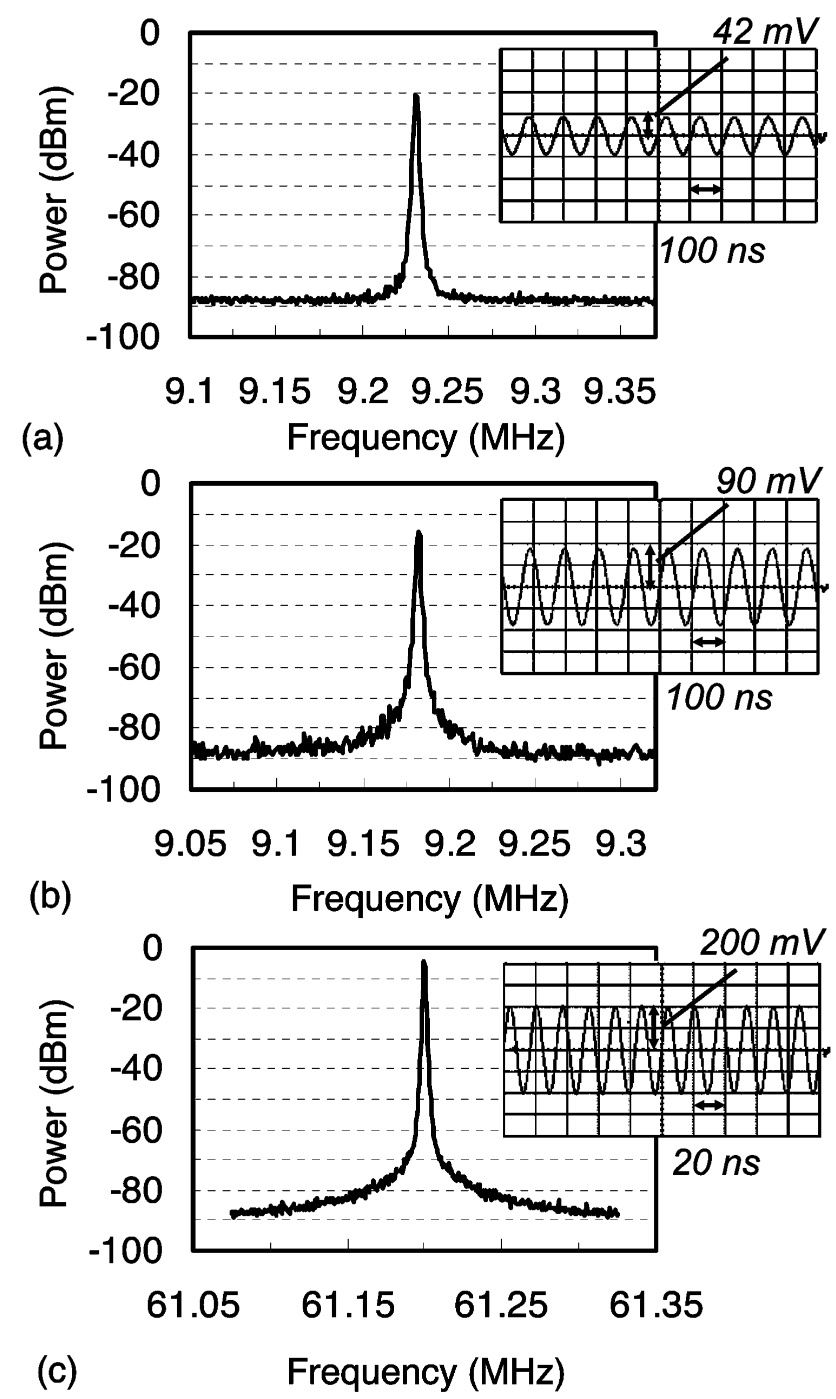
Measured Spectra and Time-Domain

- These are the measurements without using the ALC
- The oscillation self-limits due to the resonator non-linearity
- Notice the extremely small oscillation amplitudes
- With the ALC, the oscillation amplitude drops to 10mV



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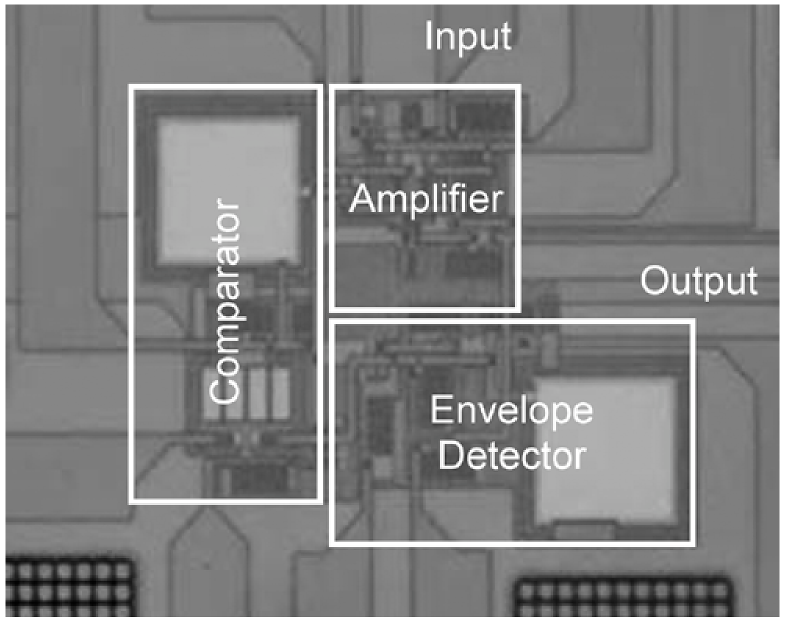
Experimental Results



- Performance close to GSM specs. DC power and area are compelling
- The measured 1/f noise much larger than expected

Integrated Circuit	Process	TSMC 0.35-μm CMOS
	Voltage Supply	± 1.65 V
	Amplifier Gain	8 kΩ
	Amplifier BW	200 MHz
	Input Resistance	2.1 kΩ
	Output Resistance	2.2 kΩ
	Power Cons. (Amp)	350 μW
	Power Cons. (ALC)	430 μW
	Layout Area	140 μm × 100 μm
10-MHz CC-Beam Resonator Oscillator	Oscillation Power	-42.2 dBm
	Phase Noise @ 1 kHz	-82 dBc/Hz
	Phase Noise @ 10 kHz	-110 dBc/Hz
	Phase Noise @ 100 kHz	-116 dBc/Hz
10-MHz Wide-Width CC-Beam Resonator Oscillator	Oscillation Power	-33.9 dBm
	Phase Noise @ 1 kHz	-80 dBc/Hz
	Phase Noise @ 10 kHz	-106 dBc/Hz
	Phase Noise @ 100 kHz	-120 dBc/Hz
60-MHz Wine-Glass Disk Resonator Oscillator	Oscillation Power	-24.6 dBm
	Phase Noise @ 1 kHz	-110 dBc/Hz
	Phase Noise @ 10 kHz	-128 dBc/Hz
	Phase Noise @ 100 kHz	-132 dBc/Hz
60-MHz ÷ 6 = 10-MHz Wine-Glass Disk Res. Oscillator	Oscillation Power	-24.6 dBm
	Phase Noise @ 1 kHz	-125 dBc/Hz
	Phase Noise @ 10 kHz	-143 dBc/Hz
	Phase Noise @ 100 kHz	-147 dBc/Hz

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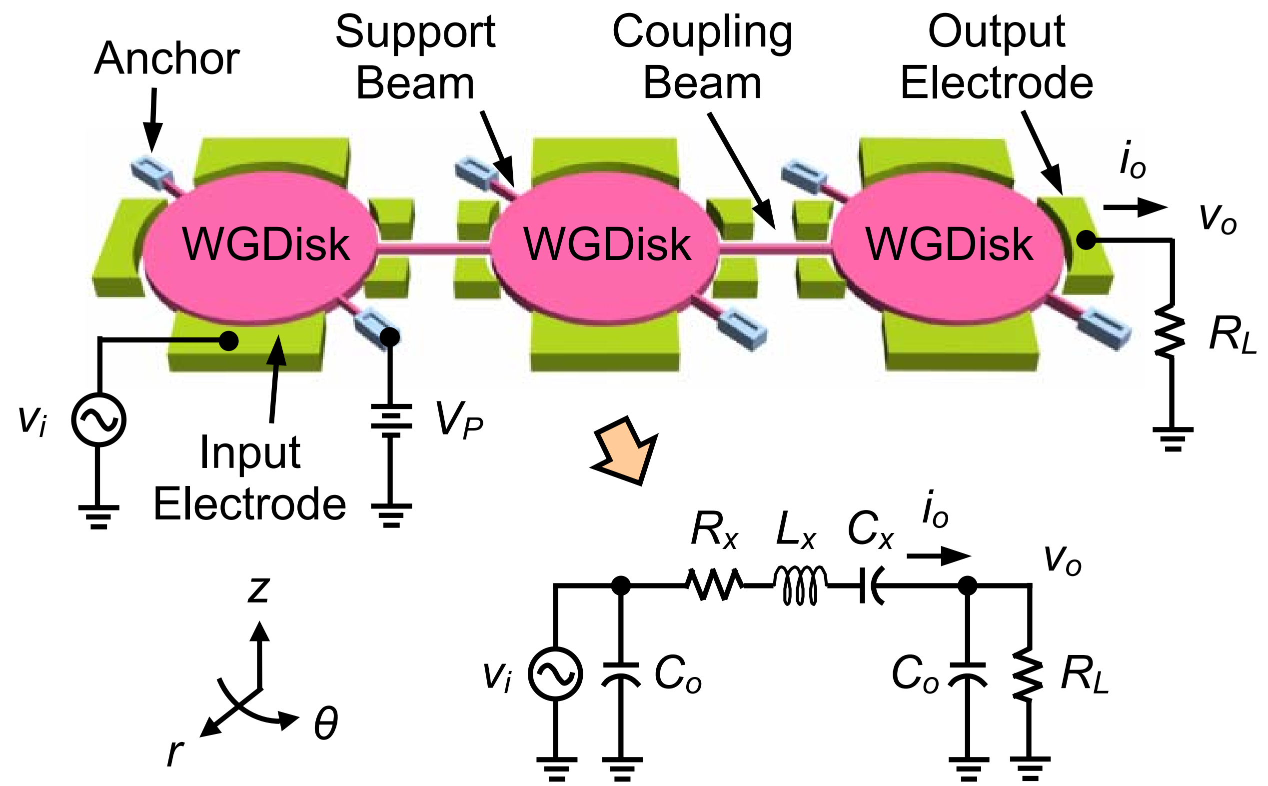
Array-Composite MEMS Wine-Glass Osc



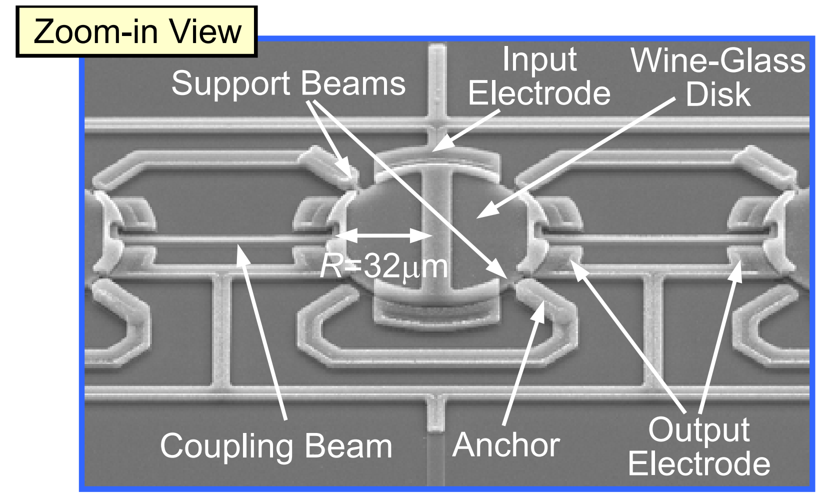
- Increase power handling capability by coupling multiple (*N*) resonators together.
- This increases power handling capability by *N*.

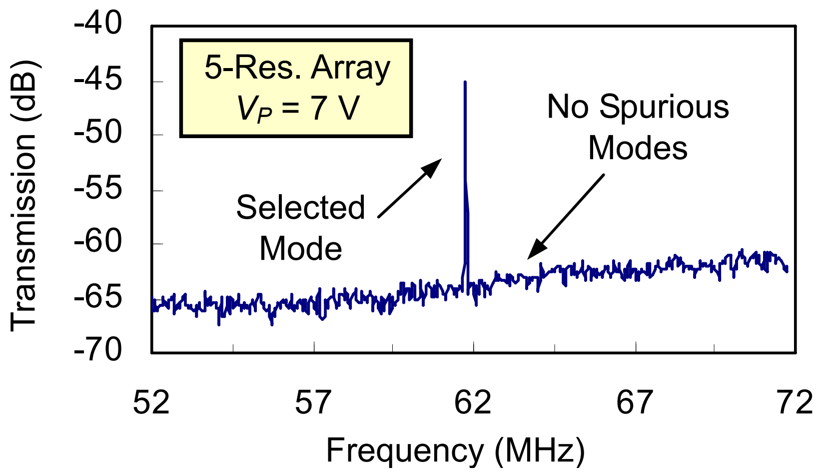
Y.-W. Lin, S.-S. Li, Z. Ren, and C. T.-C. Nguyen, "Low phase noise array-composite micromechanical wine-glass disk oscillator," *Technical Digest*, IEEE Int. Electron Devices Mtg., Washington, DC, Dec. 5-7, 2005, pp. 287-290.

Design Summary

Table 1. Oscillator Data Summary

Oscillator Design Summary		
Integrated Circuit	Process	TSMC 0.35 μm CMOS
	Voltage Supply	± 1.65 V
	Power Cons.	350 μW
	Amplifier Gain	8 kΩ
	Amplifier BW	200 MHz
	Layout Area	50 μm × 50 μm
MEMS Wine-Glass Disk Resonator Array	Process	Polysilicon-Based Surface Micromachining
	Radius, R	32 μm
	Thickness, h	3 μm
	Gap, d_o	80 nm
	Voltage Supply	10 V
	Power Cons.	~ 0 W
	Motional Resistance, R_x	5.75 kΩ, 3.11 kΩ, 1.98 kΩ, 1.25 kΩ for n = 1, 3, 5, 9
	Layout Area	n × 105 μm × 105 μm

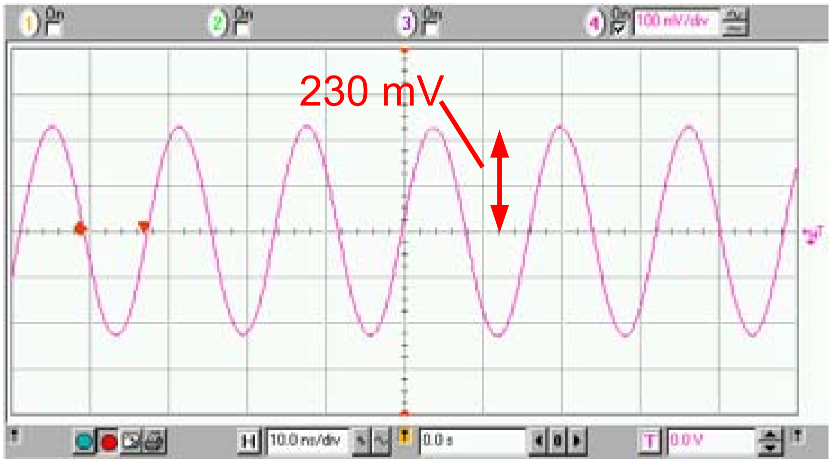


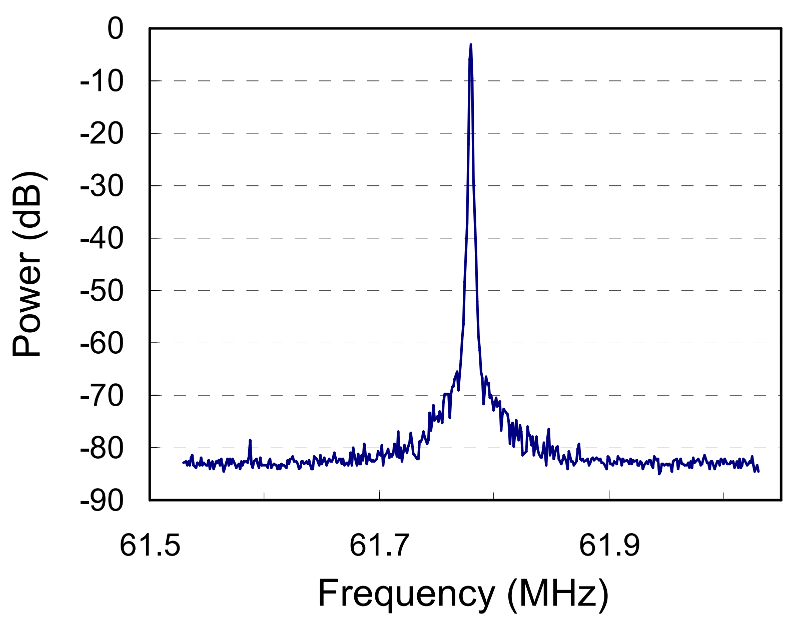


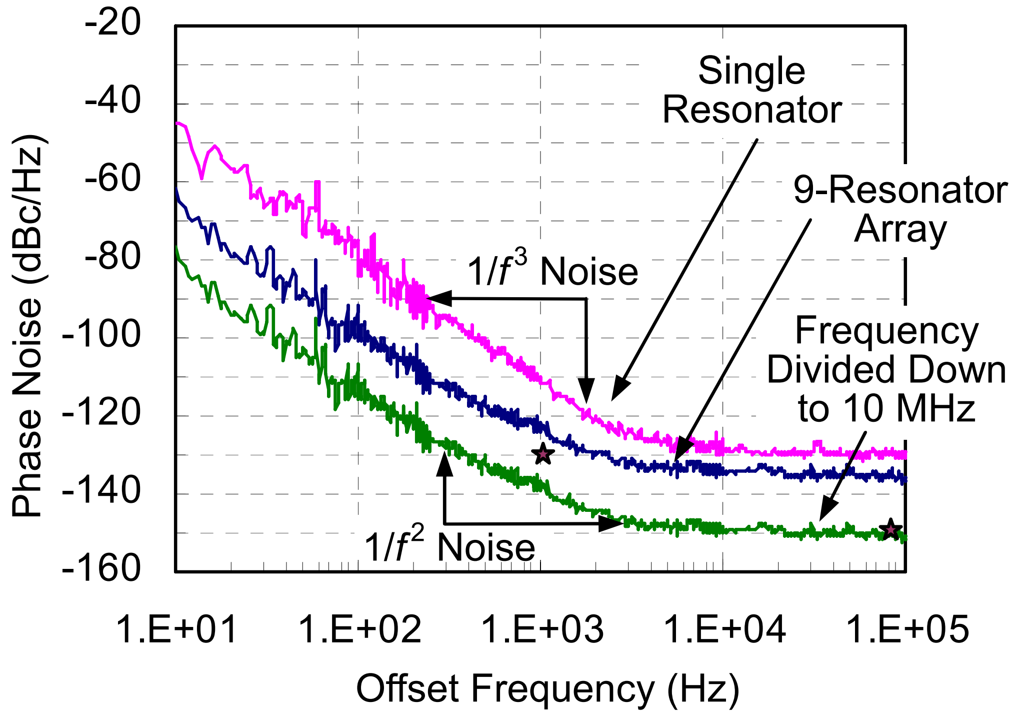
- Prototype resonator implemented in a 0.35μm CMOS process shows no spurious modes
- Area is still quite resonable compared to a bulky XTAL

Y.-W. Lin, S.-S. Li, Z. Ren, and C. T.-C. Nguyen, "Low phase noise array-composite micromechanical wine-glass disk oscillator," *Technical Digest*, IEEE Int. Electron Devices Mtg., Washington, DC, Dec. 5-7, 2005, pp. 287-290.

Measured Phase Noise



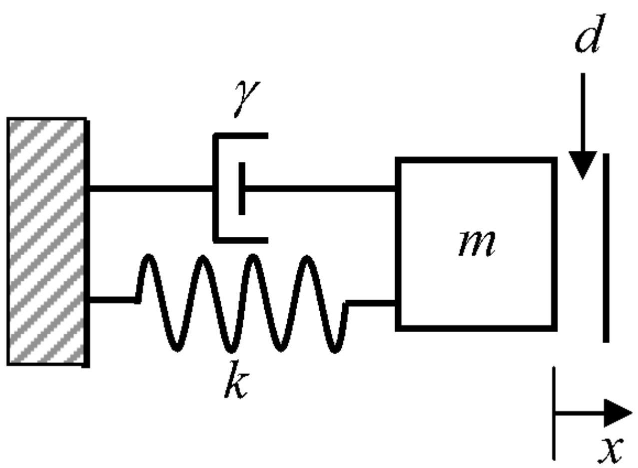


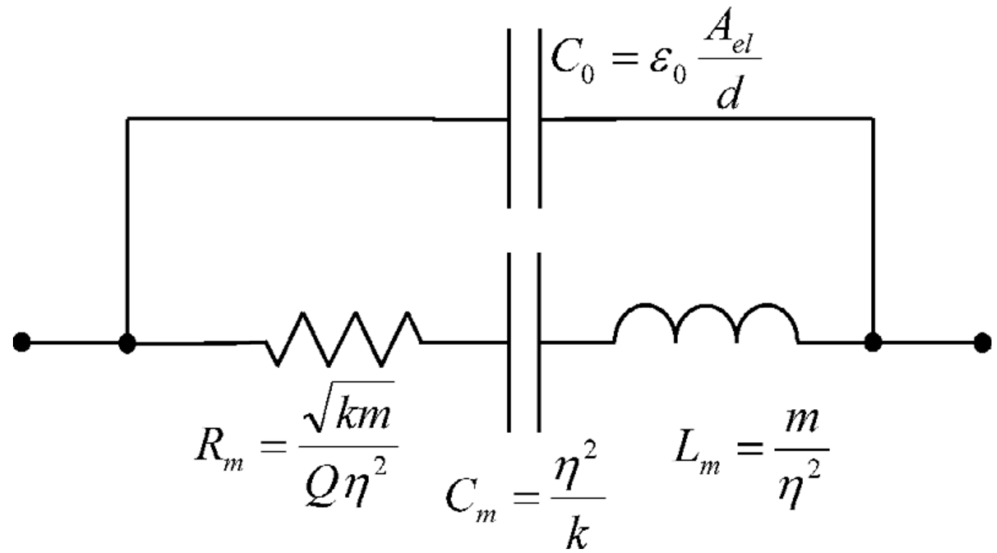


- Meets GSM specs with comfortable margin

Y.-W. Lin, S.-S. Li, Z. Ren, and C. T.-C. Nguyen, "Low phase noise array-composite micromechanical wine-glass disk oscillator," *Technical Digest*, IEEE Int. Electron Devices Mtg., Washington, DC, Dec. 5-7, 2005, pp. 287-290.

Phase Noise: Model for Resonator





$$x = H(\omega)F_e$$

$$H(\omega) = \frac{k^{-1}}{1 - \omega^2/\omega_0^2 + i\omega/Q\omega_0}.$$

$$F_e = \frac{1}{2}\frac{\partial C}{\partial x}(U_{dc} + u_{ac})^2$$

$$C = \epsilon_0 \frac{A_{el}}{d - x},$$

$$i_{sig} = \frac{\partial CU}{\partial t} \approx \frac{\partial C}{\partial t}U_{dc} + C_0\frac{\partial u_{ac}}{\partial t},$$

$$i_m \approx \eta \dot{x},$$
$$F_e \approx \eta u_{ac},$$

$$\eta = U_{dc}\frac{\partial C}{\partial x} \approx U_{dc}\frac{C_0}{d}$$

- The system is non-linear due to the electrostatic mechanism and the mechanical non-linearities

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Non-Linear Spring Constant

$$F = \frac{U_{dc}^2}{2}\frac{\partial C}{\partial x}.$$

$$k_e(x) = k_{0e}(1 + k_{1e}x + k_{2e}x^2)$$

$$k_{0e} = -\frac{U_{DC}^2 C_0}{d^2},\ k_{1e} = \frac{3}{2d},\ \text{and}\ k_{2e} = \frac{2}{d^2}.$$

- The second-order correction in the spring constant dominates
- Electrostatic non-linearity limits the drive level at high vibration amplitudes.
- The system can become chaotic at high drive amplitudes. The critical amplitude before a bifurcation is given by

$$x_c = \frac{2}{\sqrt{3\sqrt{3}Q|\kappa|}}, \qquad \kappa = \frac{3k_{2e}k_{0e}}{8k} - \frac{5k_{1e}^2 k_{0e}^2}{12k^2}.$$

$$i_m^{\max} = \eta\omega_0 x_c.$$

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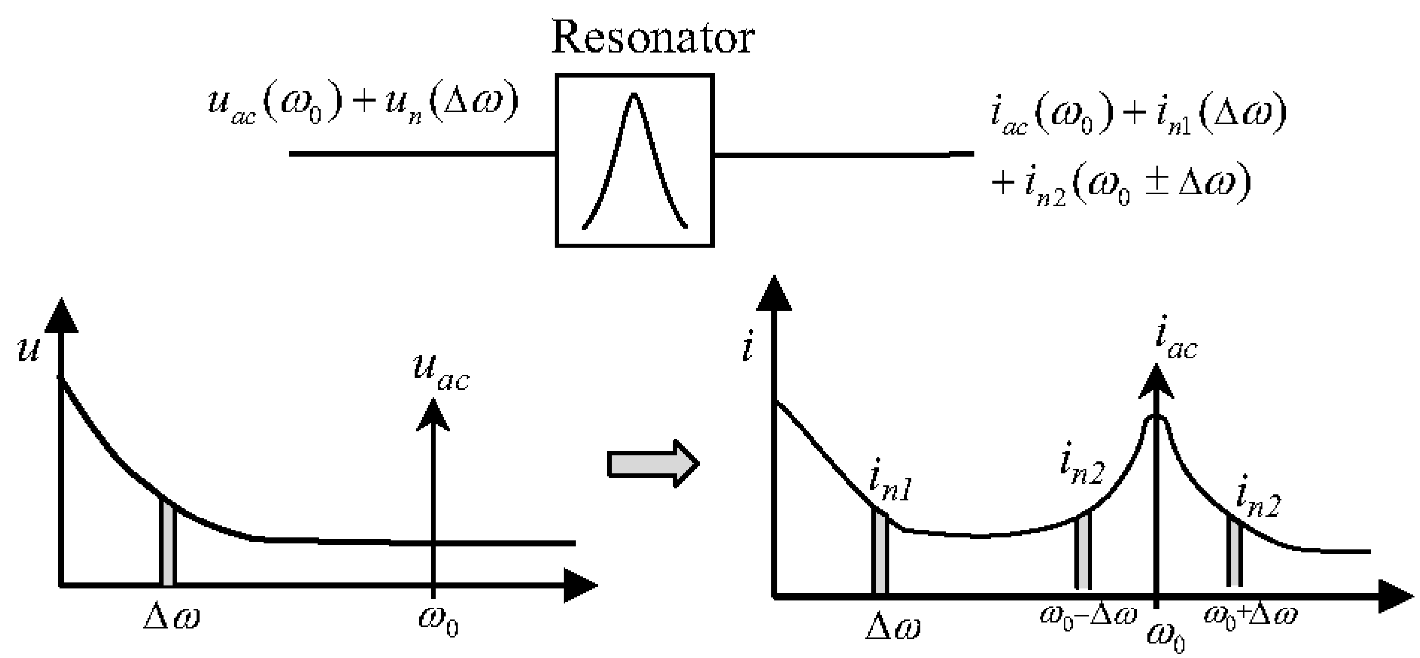
Noise Aliasing in Resonators



- As we have learned in our phase noise lectures, 1/f noise can alias to the carrier through time-varying and non-linear mechanisms. Since 1/f noise is high for CMOS, this is a major limitation

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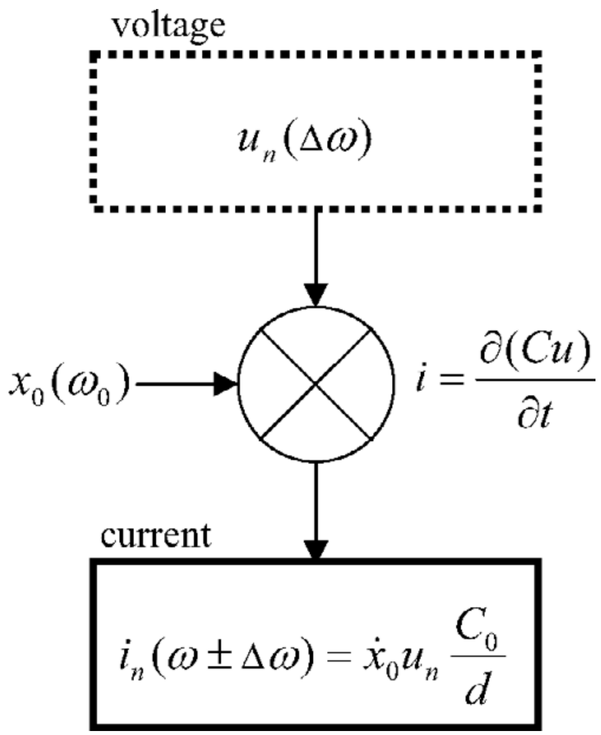
Mixing: Capacitive Current Non-Linearity



$$C(x) \approx C_0 \left(1 + \frac{x_0}{d}\right)$$

$$i_n = \frac{\partial(C(x)u_n)}{\partial t} \approx \frac{C_0}{d}\dot{x}_0 u_n + C_0 \dot{u}_n.$$

$$i_n^c = 2\Gamma_c u_{ac} u_n, \quad \Gamma_c = \frac{Q\omega_0\eta^2}{2kU_{dc}}.$$

- This term is usually much smaller (by 10X ~ 100X) than mixing due to the force non-linearity

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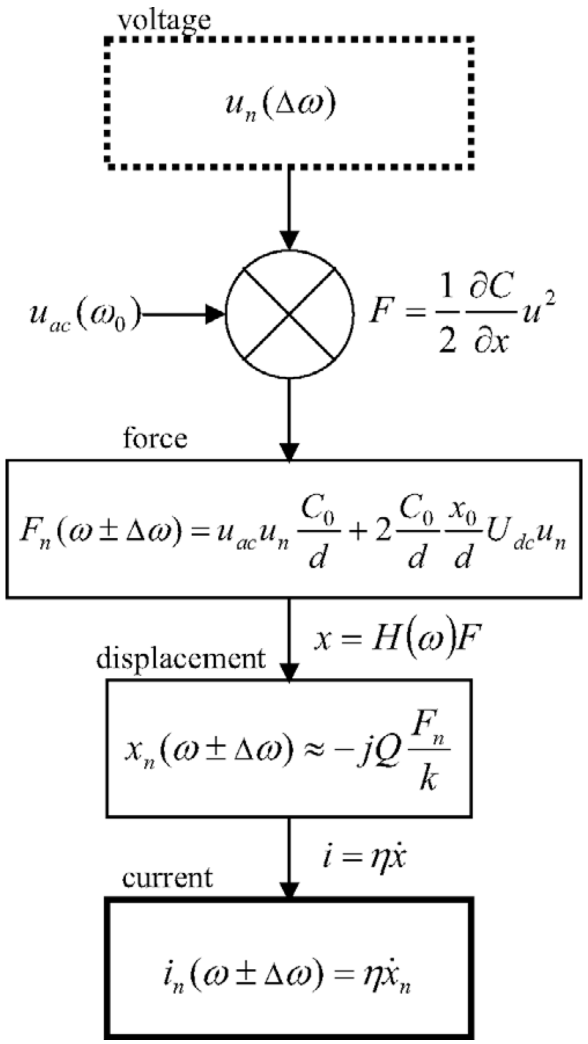
Mixing: Capacitive Force Non-Linearity



$$F_n = \frac{U^2}{2}\frac{\partial C}{\partial x} \approx \frac{(U_{dc} + u_{ac} + u_n)^2}{2}\frac{C_0}{d}\left(1 + 2\frac{x_0}{d}\right)$$

$$F_n(\omega_0 \pm \Delta\omega) \approx \frac{C_0}{d}u_{ac}u_n + 2\frac{C_0}{d}\frac{x_0}{d}U_{dc}u_n$$

$$i_n^F = 2\Gamma_F u_{ac}u_n$$

$$\Gamma_F \approx \frac{Q\omega_0\eta^2}{2kU_{dc}}\left(1 - j2\frac{Q\eta U_{dc}}{kd}\right)$$

- The form is the same as the capacitance non-linearity, but the magnitude is much higher and dominates for most resonators. A linear coupling capacitor has much reduced noise up-conversion

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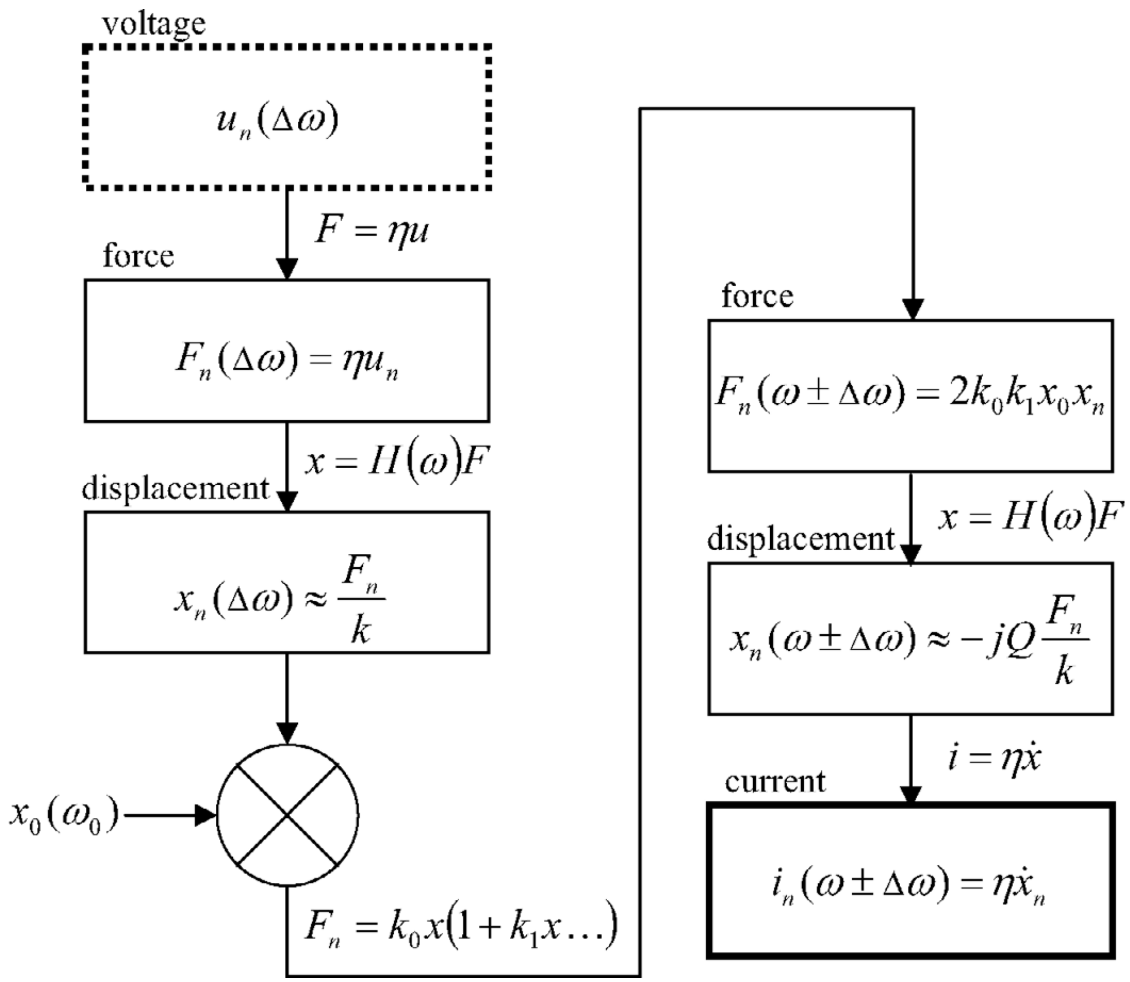
Mixing: Non-Linear Spring Force



$$x_n = H(\omega)F_n \approx \frac{\eta u_n}{k}$$

$$F_n^k = 2k_0k_1x_0x_n.$$

$$i_n^k = 2\Gamma_k u_{ac} u_n,$$

$$\Gamma_k = j\frac{3Q^2\omega_0\eta^4U_{dc}}{2d^2k^3}.$$

- Amplitude of noise at low-frequency is very small due to resonator Q. The noise is up-converted through the spring non-linearity.
- This term is the smallest of the three, about 500X smaller than the capacitance non-linearity.

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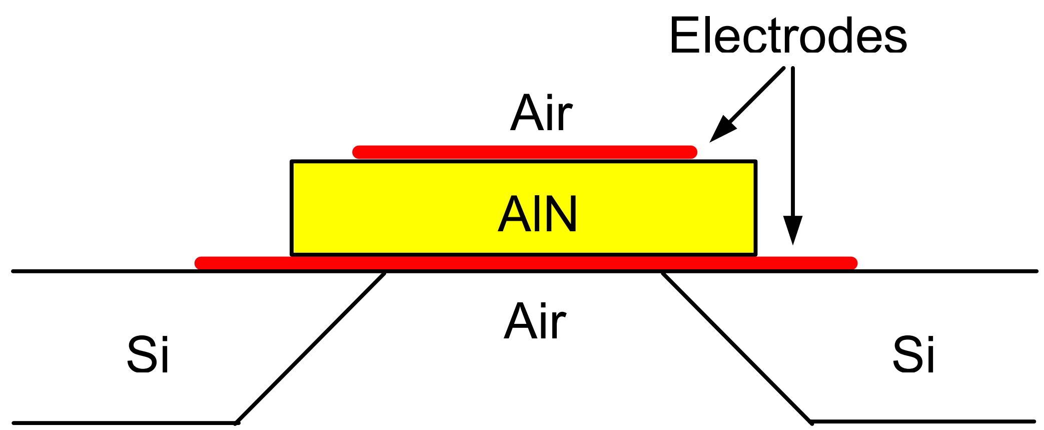
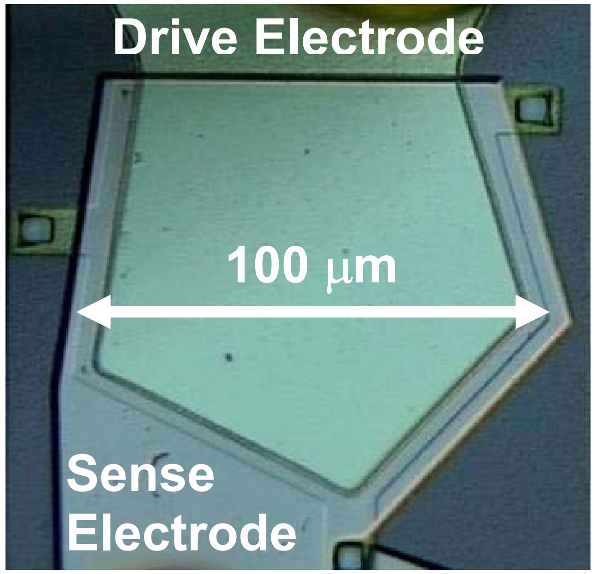
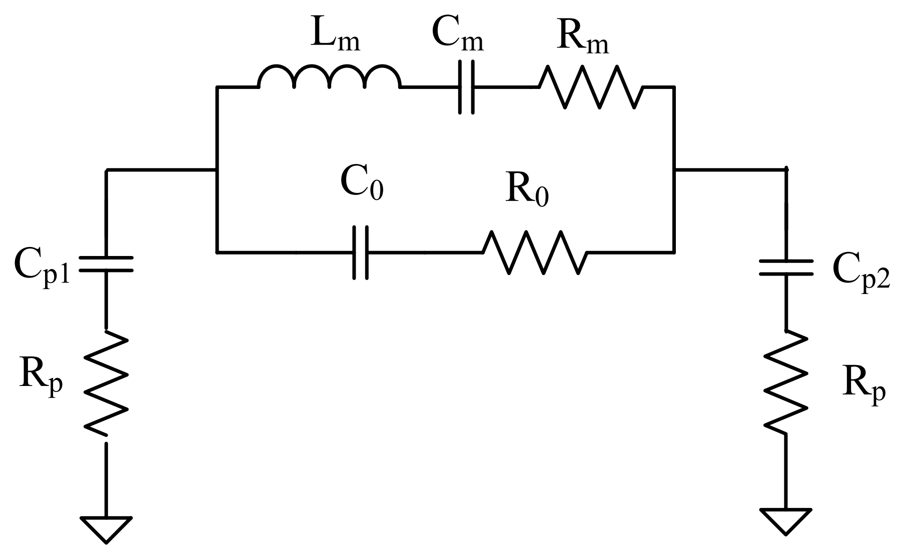
FBAR Resonator

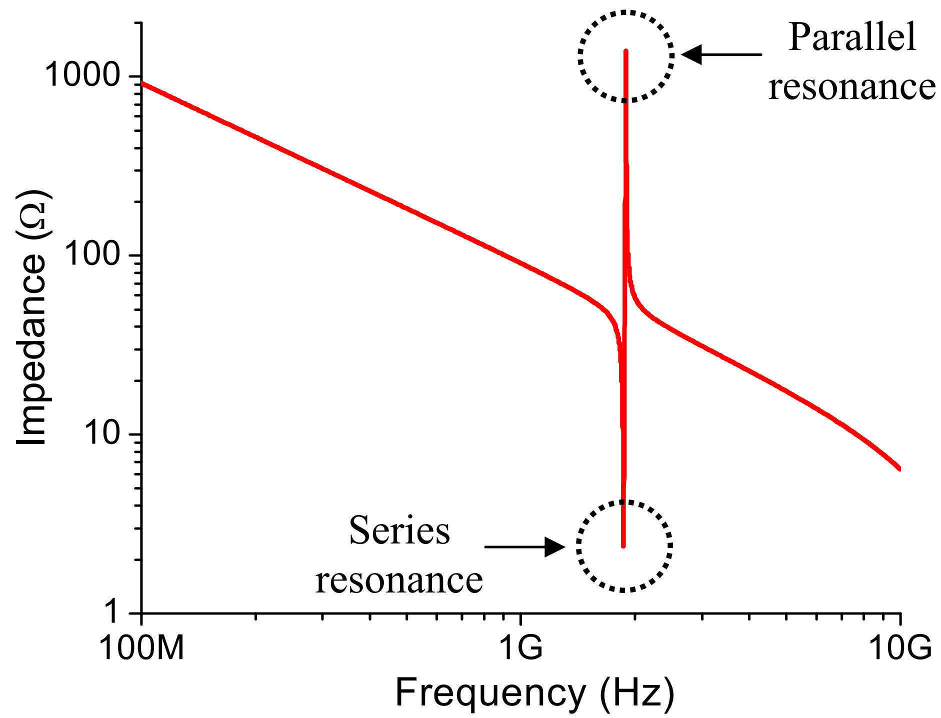




- Another "MEMS" technology is the Thin *F*ilm *B*ulk Wave *A*coustic *R*esonators (FBAR)
- It uses a thin layer of Aluminum-Nitride piezoelectric material sandwiched between two metal electrodes
- The FBAR has a small form factor and occupies only about 100μm x 100μm.

FBAR Resonance

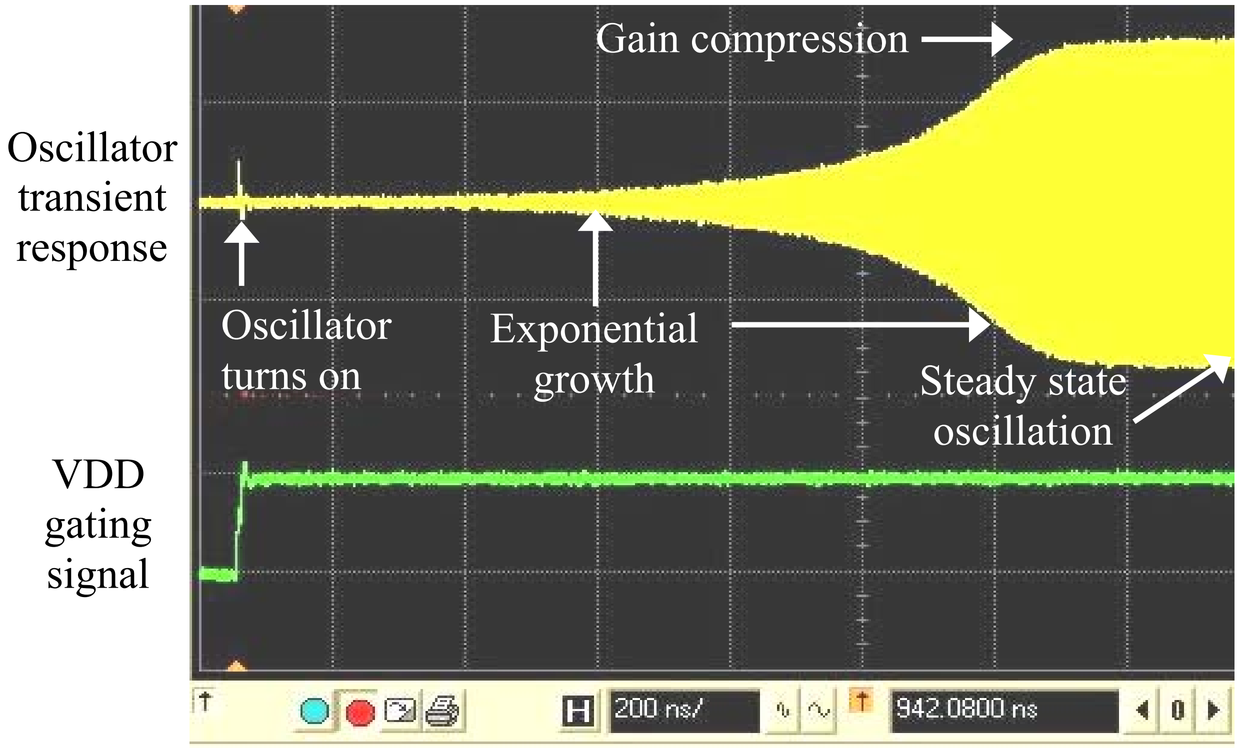




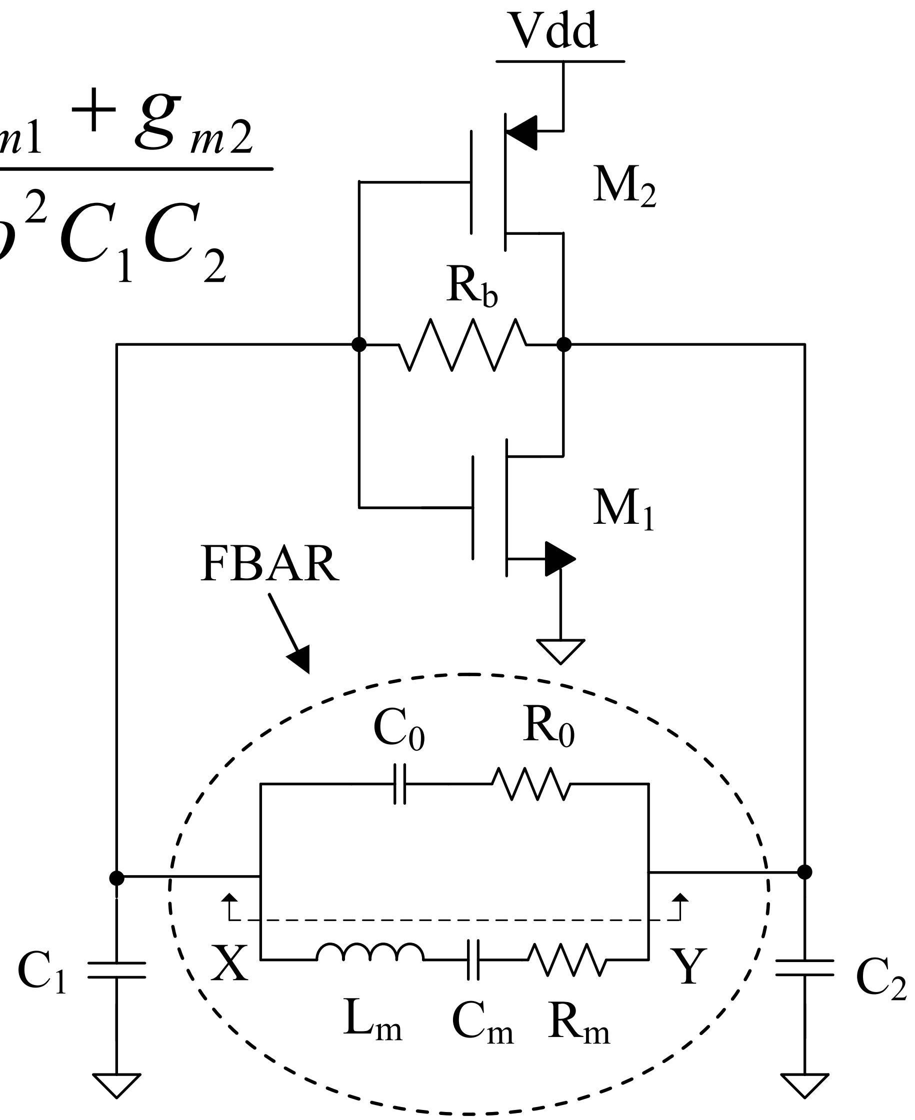
- Very similar to a XTAL resonator. Has two modes: series and parallel
- Unloaded Q ~ 1000
- This technology will not be integrated directly with CMOS, but there is a potential for advanced packaging or procesing.

FBAR Oscillator

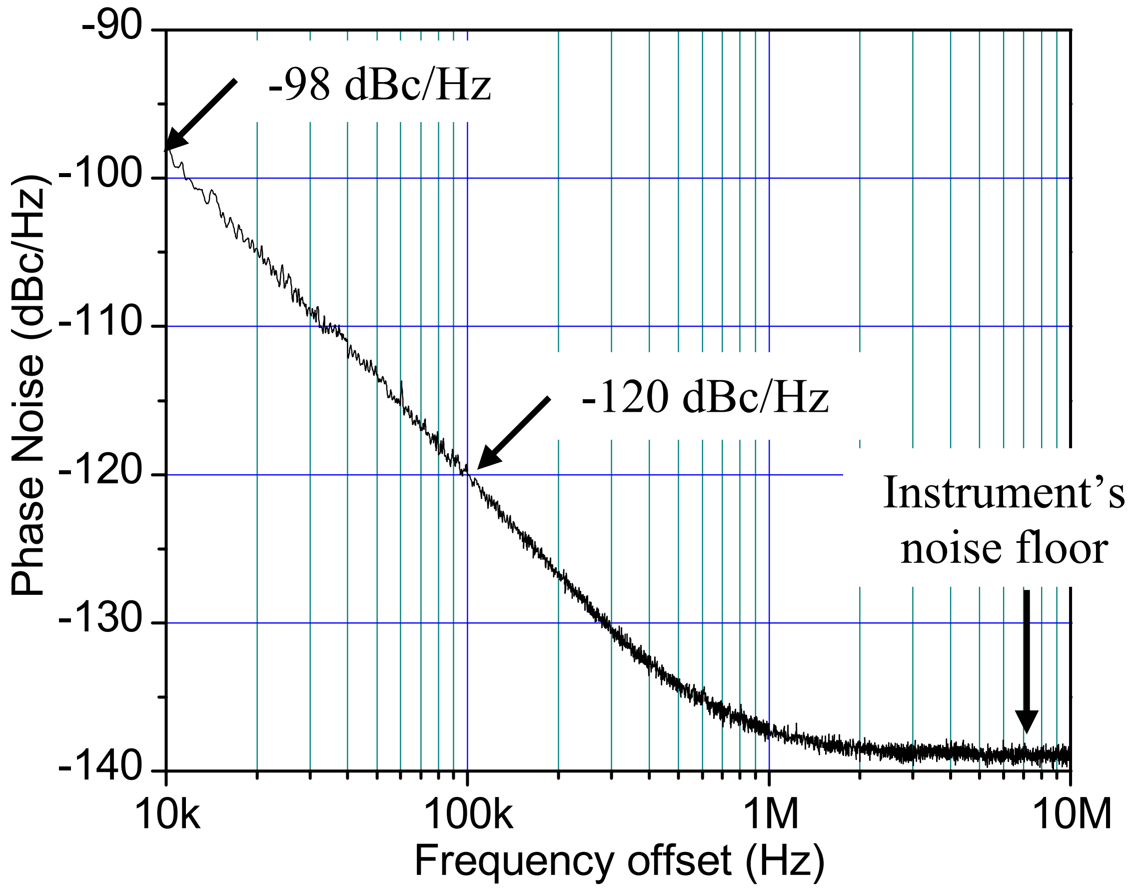
- Rm ~ 1 ohm
- gm ~ 7.8 mS used (3X)
- C1=C2=.7pF
- gm/Id ~ 19, Id ~ 205μA
- Start-up behavior shown below:

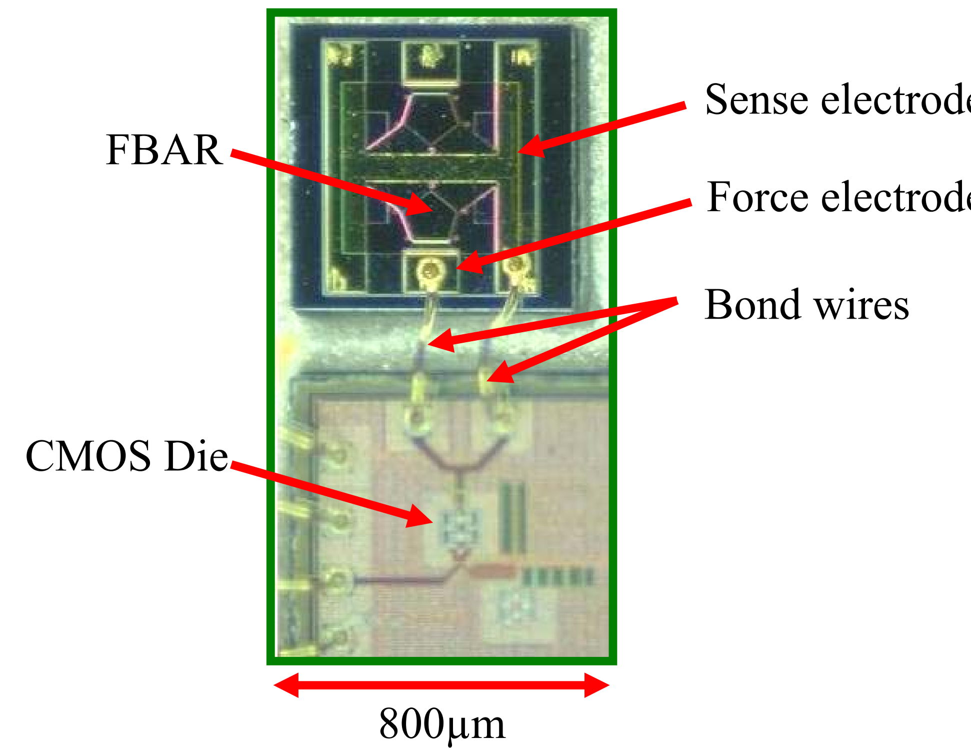


$$-\frac{g_{m1}+g_{m2}}{\omega^2 C_1 C_2}$$



Measured Results on FBAR Osc





- Operate oscillator in "current limited" regime
- Voltage swing ~ 167 mV, Pdc ~ 104 μW