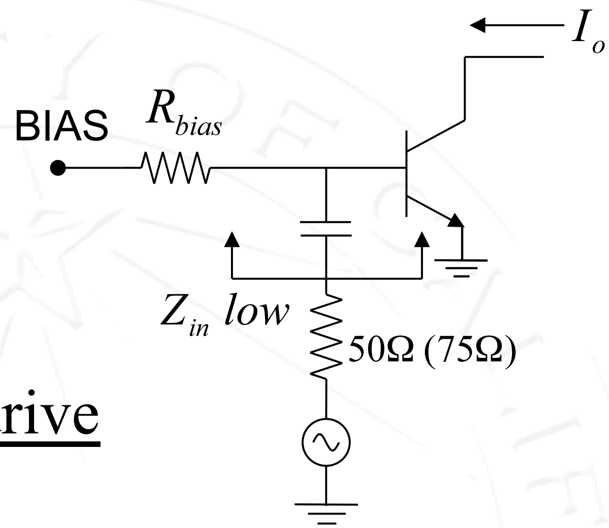
EECS 242:
BJT High Frequency Distortion

Professor Ali M Niknejad
Advanced Communication Integrated Circuits

University of California, Berkeley

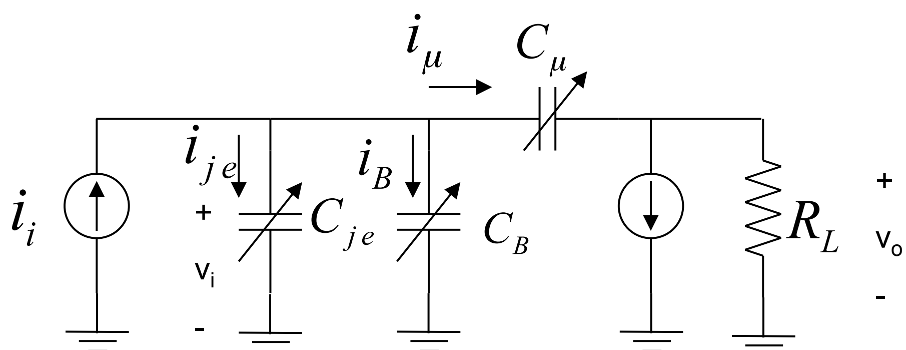
High Frequency Distortion in BJTs

Meyer:



Assume current drive

Large Circuit Equivalent Circuit:



$$i_c = I_Q\left(e^{v_1/V_T} - 1\right)$$

neglect

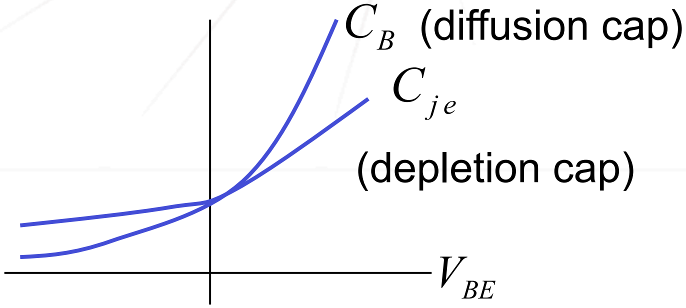
r_π, r_b, r_o

Distortion Due to Diffusion Capacitor

C_B is a non-linear diffusion capacitor

$$C_B = C_\pi - C_{je}$$

$$Q_B = \tau_F I_C \qquad C_B = \frac{dQ_B}{dV_{BE}} = \tau_F \frac{dI_C}{dV_{BE}} = \tau_F \frac{qI_C}{kT} = \tau_F \frac{I_S}{V_T} e^{V_{BE}/V_T}$$



C_{je} is emitter -base depletion capacitor

Assume constant

$$C_{je} \propto \frac{1}{(\varphi + V_{BE})^{\frac{1}{n}}}$$

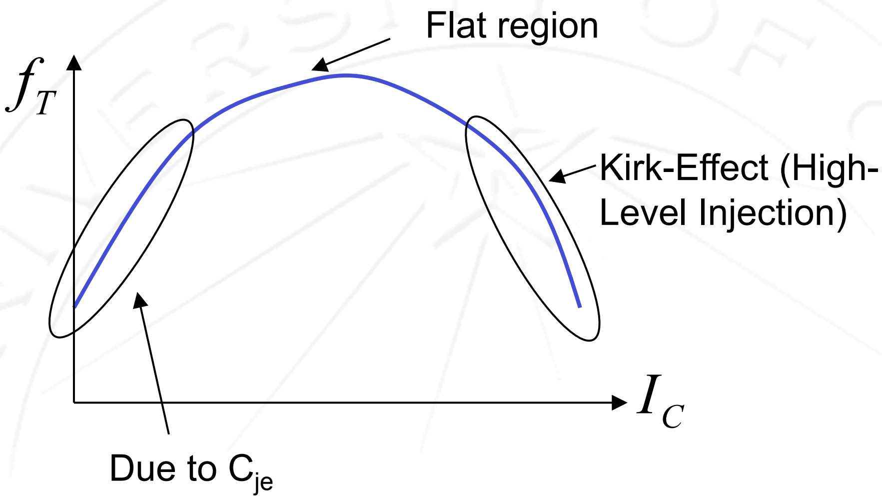
If f_T of a BJT is constant with I_C, then we have no high frequency distortion in the device for current drive and $C_B >> C_{je}$

$$i_i = \tau_F \frac{di_c}{dt}$$

$\Rightarrow$ Linear Differential Equation

f_T constant $\Rightarrow \tau_F$ constant

Kirk Effect



$$i_C = I_C - I_Q = I_Q\left(e^{v_1/V_T} - 1\right)$$

$$i_i = i_x + i_b + i_\mu$$

$$= C_{je}\frac{dv_1}{dt} + \frac{dQ_B}{dt} + \frac{dQ_\mu}{dt}$$

$$\frac{dQ_\mu}{dt} = \underbrace{\frac{dQ_\mu}{d(v_1 - v_o)}}_{C_\mu(v)}\frac{d(v_1 - v_o)}{dt} = \underbrace{C_\mu\frac{dv_1}{dt}}_{0} - C_\mu\frac{dv_o}{dt}$$

$$C_\mu = \frac{k}{(\phi + V_{CB})^{\frac{1}{n}}} = \frac{k}{(\phi + V_{CBQ} + V_{CB})^{\frac{1}{n}}}$$

Governing Differential Eq.

$$\frac{dQ_B}{dt} = \tau_F \frac{dI_c}{dt} = \tau_F \frac{di_c}{dt}$$

$$i_i = C_{je}\frac{dv_1}{dt} + \tau_F \frac{di_c}{dt} - C_\mu \frac{dv_o}{dt}$$

$$v_1 = V_T \ln\left(\frac{i_c + I_Q}{I_Q}\right)$$

$$\frac{dv_1}{dt} = \frac{dv_1}{di_c}\frac{di_c}{dt} = \frac{V_T}{\cancel{I_Q}}\frac{\cancel{I_Q}}{i_c + I_Q}\frac{di_c}{dt} \quad = \frac{V_T}{i_c + I_Q}\frac{di_c}{dt}$$

$$\therefore \quad i_i = C_{je}\frac{V_T}{i_c + I_Q}\frac{di_c}{dt} + \tau_F \frac{di_c}{dt} - C_\mu \frac{dv_o}{dt}$$

BJT Series Expansion

$$v_o = -i_c R_L \qquad i_i = -\{\frac{1}{R_L}\left(\tau_F + \frac{C_{je}V_T}{I_Q + i_c}\right) + C_\mu\}\frac{dv_o}{dt}$$

$$= -\frac{1}{R_L}[\tau_F + \frac{C_{je}V_T}{\underbrace{I_Q + i_c}} + C_\mu R_L]\frac{dv_o}{dt}$$

$$\frac{1}{I_Q\left(1 + \frac{i_c}{I_Q}\right)} \Rightarrow \frac{1}{1+x} = 1 - x + x^2 + \ldots$$

$$C_\mu = \frac{C_{\mu_o}}{\left(1 + \frac{v_o}{V_Q}\right)^{\frac{1}{n}}} = C_{\mu_o}\left(1 - \frac{1}{n}\frac{v_o}{V_Q} + \frac{1}{2n}\left(\frac{1}{n} + 1\right)\left(\frac{v_o}{V_Q}\right)^2 + \ldots\right)$$

BJT Series Expansion (cont)

$$\therefore \quad i_i \cong -\frac{1}{R_L}[\tau_F + \frac{C_{je}V_T}{I_Q} - \frac{C_{je}V_T}{I_Q}\frac{i_c}{I_Q} + \frac{C_{je}V_T}{I_Q}\left(\frac{i_c}{I_Q}\right)^2 +$$

$$C_{\mu_o}R_L - C_{\mu_o}R_L\frac{1}{n}\frac{v_o}{V_Q} + C_{\mu_o}R_L\frac{1}{2n}\left(1+\frac{1}{n}\right)\left(\frac{v_o}{V_Q}\right)^2]\times\frac{dv_o}{dt}$$

Substitute $v_o = -i_c R_L$

$$i_i \cong \left(a_1 + a_2 v_o + a_3 v_o^{\ 2}\right)\frac{dv_o}{dt} \quad (*)$$

$$\cong a_1\frac{dv_o}{dt} + \frac{1}{2}a_2\frac{dv_o^{\ 2}}{dt} + \frac{1}{3}a_3\frac{dv_o^{\ 3}}{dt} \quad (**)$$

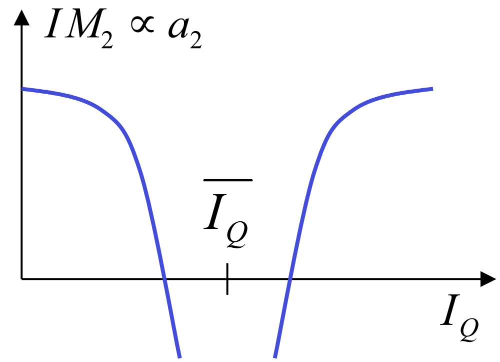
Memoryless Terms

Non-linear differential equation for v_o vs. i_i where

$$a_1 = -\frac{1}{R_L}\left(\tau_F + \frac{C_{je}V_T}{I_Q} + C_{\mu_o}R_L\right)$$

$$a_2 = -\frac{1}{R_L}\left(\frac{C_{je}V_T}{R_L I_Q^{\,2}} - C_{\mu_o}R_L\frac{1}{n}\frac{1}{V_Q}\right)$$

null



$$a_3 = -\frac{1}{R_L}\left(\frac{C_{je}V_T}{R_L^{\,2} I_Q^{\,3}} + C_{\mu_o}R_L\frac{1}{2n}\left(1+\frac{1}{n}\right)\frac{1}{V_Q^{\,2}}\right)$$

always add

Memory Terms

In (**), put

$$v_o = A_1(j\omega_a)\circ i_i + A_2(j\omega_a, j\omega_b)\circ i_i^2 + \ldots$$

$$i_i = a_1 \frac{dv_o}{dt} + \frac{1}{2} a_2 \frac{dv_o^2}{dt} + \frac{1}{3} a_3 \frac{dv_o^3}{dt}$$

<u>First Order</u>:

$$i_i = a_1 j\omega A_1(j\omega)\circ i_i$$

$$\therefore \quad A_1(j\omega) = \frac{1}{j\omega a_1}$$

Second Order Memory

Second Order:

$$0 = a_1 j(\omega_a + \omega_b)A_2(j\omega_a, j\omega_b) + \frac{1}{2} a_2 (j\omega_a + j\omega_b) A_1(j\omega_a) A_1(j\omega_b)$$

$$\therefore \quad A_2(j\omega_a, j\omega_b) = -\frac{1}{2}\frac{a_2}{a_1} A_1(j\omega_a) A_1(j\omega_b)$$

$$A_2(j\omega_a, j\omega_b) = -\frac{1}{2}\frac{a_2}{a_1}\frac{1}{j\omega_a a_1}\frac{1}{j\omega_b a_1}$$

$$= \frac{1}{2}\frac{a_2}{a_1^{\ 3}}\frac{1}{\omega_a \omega_b}$$

Third Order Memory

Third Order:

$$0 = [a_1 A_3 + \frac{1}{2} a_2 2\overline{A_1 A_2} + \frac{1}{3} a_3 A_1 A_1 A_1](j\omega_a + j\omega_b + j\omega_c)$$

$$\therefore \quad A_3 = \frac{-\frac{1}{2} a_2 2\overline{A_1 A_2} + \frac{1}{3} a_3 A_1 A_1 A_1}{a_1}$$

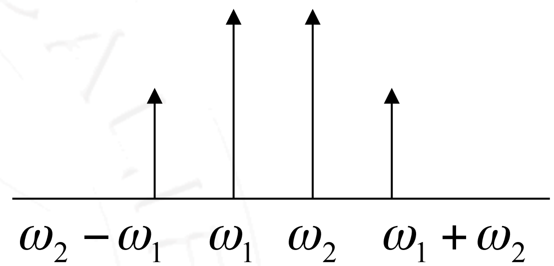
$$A_3 = \frac{\frac{a_3}{3} - \frac{a_2^{\,2}}{2a_1}}{j\omega_a \omega_b \omega_c a_1^{\,4}}$$

No null because a_3 is negative

IM_2 in BJT at High Frequency

$$i_i = I_1 \cos\omega_1 t + I_2 \cos\omega_2 t$$

$$IM_2 = \frac{|A_2| I_1 I_2}{\hat{v}_o} \frac{2!}{1!1!} \frac{1}{2}$$



$$\hat{v}_o = |A_1(j\omega_1)| I_1 = |A_1(j\omega_2)| I_2$$

$$\therefore \quad IM_2 = \frac{|A_2(j\omega_1, \pm j\omega_2)|}{|A_1(j\omega_1)||A_1(j\omega_2)|} \hat{v}_o$$

$$IM_2 = \frac{1}{2} \frac{a_2}{a_1^3} \frac{1}{\omega_1 \omega_2} a_1 \omega_1 a_1 \omega_2 \hat{v}_o = \frac{1}{2} \frac{a_2}{a_1} \hat{v}_o$$

Independent of frequency at high frequency

IM3 in BJT at High Frequency

$$IM_3 = \frac{3!}{2!1!}\frac{1}{2^2}\frac{I_1 I_o^{\ 2}|A_3|}{\hat{v}_o}$$

Again $\quad |A_1(j\omega_1)|I_1 = |A_1(j\omega_2)|I_2 = \hat{v}_o$

$$IM_3 = \frac{3}{4}\frac{|A_3(j\omega_2, j\omega_2, -j\omega_1)|}{|A_1(j\omega_1)||A_1(j\omega_2)||A_1(j\omega_2)|}\hat{v}_o^{\ 2} = \frac{3}{4}\hat{v}_o^{\ 2}\frac{\frac{a_3}{3} - \frac{a_3^{\ 2}}{2a_1}}{a_1} \approx \frac{3}{4}\hat{v}_o^{\ 2}\frac{a_3}{3a_1}$$

$$a_1 = -\frac{1}{R_L}\left(\tau_F + \frac{C_{je}}{g_m Q} + C_{\mu_o} R_L\right)$$

$$a_3 = -\frac{1}{R_L}\left(\frac{C_{je} V_T}{R_L^{\ 2} I_Q^{\ 3}} + C_{\mu_o} R_L \frac{1}{2n}\left(\frac{1}{n} + 1\right)\frac{1}{V_Q^{\ 2}}\right)$$

Example: 2 GHz LNA Front End

$\tau_F = 12ps$

$C_{\mu_o} = 37fF$

$r_b = 10\Omega$

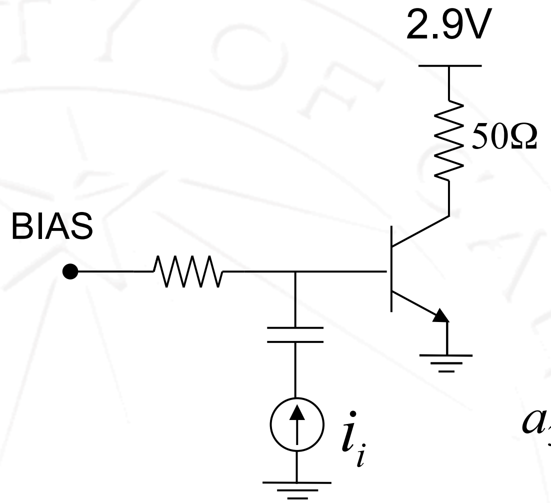
$I_C = 3mA$

$C_{je} = 416fF$

$R_L = 50\Omega$

$\eta = 3$

$V_Q = 2V$



$$Supply \approx \underbrace{V_{BE}}_{750\text{ mA}} + V_Q + \underbrace{3mA \times 50\Omega}_{150\text{ mA}}$$

= 2.9V

$$a_3 = -\frac{1}{R_L}\left(1.6\times10^{-10} + 1\times10^{-13}\right)$$

$$a_1 = -\frac{1}{R_L}\left(12 + 3.6 + 1.9\right)\times10^{-12}$$

$$IM_3 = \frac{3}{4}\hat{v}_o^{\,2}\frac{a_3}{3a_1} = 1 \quad \text{for intercept point}$$

$\hat{v}_o = 0.66V$ into 50Ω

=+ 6 dBm output IP_3

→ Very close to SPICE and measurements

LNA Example (cont)

Note:

$$r_\pi = \frac{\beta}{g_m} = \frac{150}{g_m} \approx 1.3k\Omega$$

$$C_\pi = C_{je} + C_B \qquad C_B = \tau_F g_m = 1.38pF$$

$$C_{je} = 0.416pF$$

$$\frac{1}{\omega C_\pi} = 44\Omega \leftarrow \text{Low compared to } r_\pi$$

LNA with Emitter Degen

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High-Frequency Nonlinearity Analysis of Common-Emitter and Differential-Pair Transconductance Stages

Keng Leong Fong, *Member, IEEE*, and Robert G. Meyer, *Fellow, IEEE*

- Analysis of BJT transconductor as a non-linear Gm cell
- Include emitter degeneration.

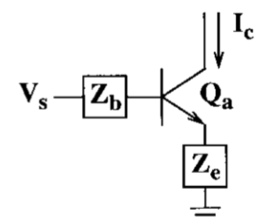


Fig. 1. Common-emitter transconductance stage.

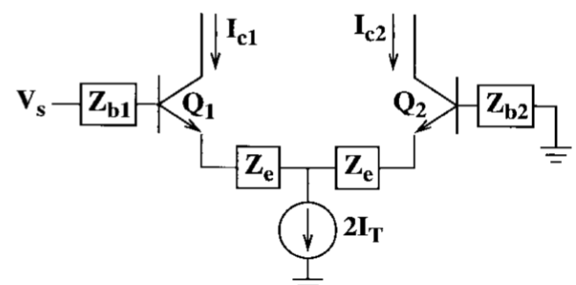


Fig. 2. Differential-pair transconductance stage.

KCL Equations

$$V_s = (sC_{je}V_\pi + s\tau_F I_c + I_c/\beta_0)(Z_b + Z_e) + I_c Z_e + V_\pi \quad (1)$$

$$I_c = A_1(s) \circ V_s + A_2(s_1,\, s_2) \circ V_s^2 + A_3(s_1,\, s_2,\, s_3) \circ V_s^3 + \cdots$$

$$V_\pi = C_1(s_1) \circ V_s + C_2(s_1,\, s_2) \circ V_s^2 + C_3(s_1,\, s_2,\, s_3) \circ V_s^3 + \cdots$$

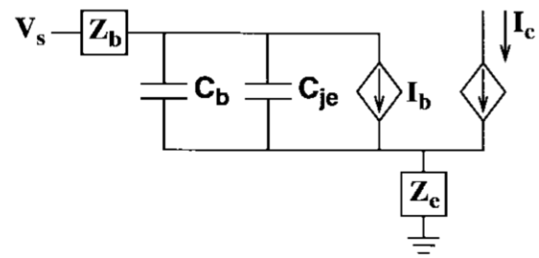


Fig. 4. Model of common-emitter transconductance stage.

$$A_1(s) = g_m C_1(s)$$

$$A_2(s_1,\, s_2) = g_m c_2(s_1,\, s_2) + \frac{I_Q}{2V_T^2} C_1(s_1) C_1(s_2)$$

$$A_3(s_1,\, s_2,\, s_3) = g_m C_3(s_1,\, s_2,\, s_3) + \frac{I_Q}{6V_T^3} C_1(s_1) C_1(s_2) C_1(s_3) + \frac{I_Q}{V_T^2} \overline{C_1 C_2}.$$

$$C_1(s) = \frac{1}{[sC_{je}Z(s) + s\tau_F g_m Z(s) + g_m Z(s)/\beta_0 + 1 + g_m Z_e(s)]}$$

$$C_2(s_1,\, s_2) = -C_1(s_1+s_2) C_1(s_1) C_1(s_2) \frac{I_Q}{2V_T^2} \times \left[(s_1+s_2)\tau_F Z(s_1+s_2) + \frac{Z(s_1+s_2)}{\beta_0} + Z_e(s_1+s_2)\right]$$

$$C_3(s_1,\, s_2,\, s_3) = -\frac{A_1(s_1+s_2+s_3) I_Q}{6V_T^3} \times \left[A_1(s_1)A_1(s_2)A_1(s_3) + 6V_T \overline{A_1 A_2}\right] \times \left[(s_1+s_2+s_3)\tau_F Z(s_1+s_2+s_3) + \frac{Z(s_1+s_2+s_3)}{\beta_0} + Z_e(s_1+s_2+s_3)\right]$$

IM3 Calculation

$$|IM_3| = \left|\frac{3}{4}\frac{A_3(s_a,\, s_a,\, -s_b)}{A_1(2s_a - s_b)}\right| |V_s|^2$$

$$\approx \left|\frac{A_1(s)}{I_Q}\right|^3 \left|\frac{V_T}{4}[1 + sC_{je}Z(s)]\right.$$

$$\times \left\{-1 + \frac{A_1(\Delta s)}{g_m}[1 + \Delta s C_{je} Z(\Delta s)]\right.$$

$$\left. + \frac{A_1(2s)}{2g_m}[1 + 2sC_{je}Z(2s)]\right\}\Bigg| |V_s|^2$$

These terms can resonate:

$$[1 + sC_{je}Z_b(s) + sC_{je}Z_e(s)].$$

For small offset frequencies:

$$[1 + \Delta s C_{je} Z(\Delta s)] \approx 1$$

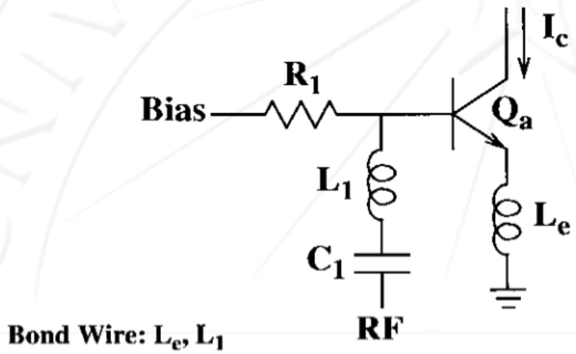
$$Z(s) = Z_b(s) + Z_e(s)$$

$$|IM_3| \approx \left|\frac{A_1(s)}{I_Q}\right|^3 \left|\frac{V_T}{4}[1 + sC_{je}Z(s)]\right.$$

$$\times \left\{-1 + \frac{A_1(\Delta s)}{g_m}\right.$$

$$\left. + \frac{A_1(2s)}{2g_m}[1 + 2sC_{je}Z(2s)]\right\}\Bigg| |V_s|^2$$

Typical Example



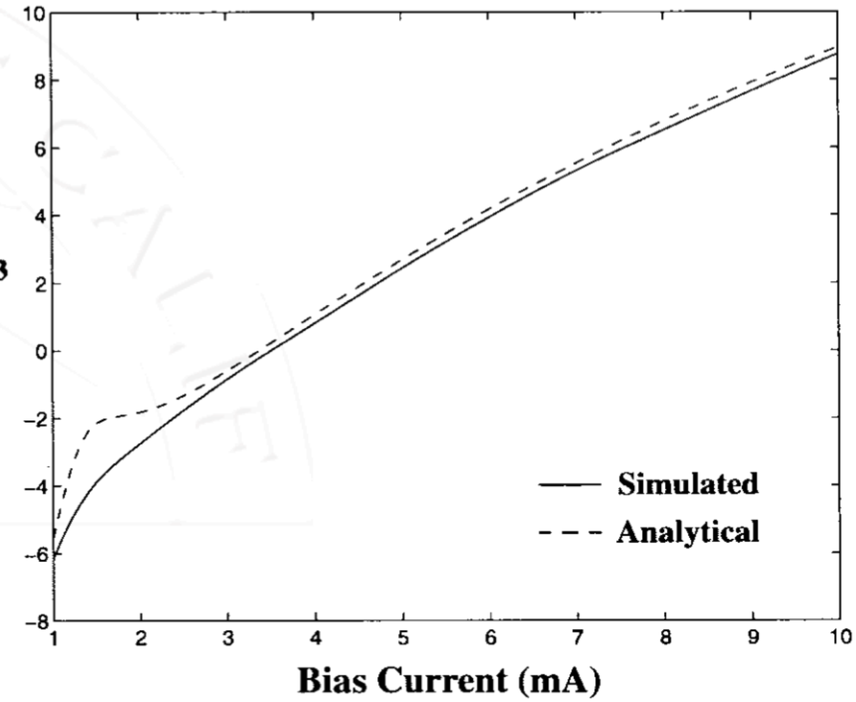


Fig. 6. Input IP3 versus bias current.

significant errors. The two RF sinusoidal signals used are at 900 and 910 MHz, respectively. The component values used are: $\tau_F = 10.5\,\text{ps}, C_{je} = 1.17\,\text{pF}, \beta = 73, L_e = 2.4\,\text{nH}, L_1 = 3.5\,\text{nH}, C_1 = 20\,\text{pF}$, and $R_1 = 150\,\Omega$.

References

- **UCB EECS 242 Class Notes**, Robert G. Meyer, Spring 1995
- K. L. Fong and R. G. Meyer, "High-Frequency Nonlinearity Analysis of Common-Emitter and Differential Pair Transconductance Stages," *JSSC*, pp. 548-555, vol. 33, no. 4, April 1998.