Crystal and MEMS Oscillators (XTAL)

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Quartz crystal is a piezoelectric material. An electric field causes a mechanical displacement and vice versa. Thus it is a electromechanical transducer.

The equivalent circuit contains series $LCR$ circuits that represent resonant modes of the XTAL. The capacitor $C_0$ is a physical capacitor that results from the parallel plate capacitance due to the leads.
Acoustic waves through the crystal have phase velocity $\nu = 3 \times 10^3 \text{ m/s}$. For a thickness $t = 1 \text{ mm}$, the delay time through the XTAL is given by

$$\tau = \frac{t}{\nu} = \frac{10^{-3} \text{ m}}{3 \times 10^3 \text{ m/s}} = \frac{1}{3} \mu\text{s}.$$ 

This corresponds to a fundamental resonant frequency

$$f_0 = \frac{1}{\tau} = \frac{\nu}{t} = 3 \text{ MHz} = \frac{1}{2\pi \sqrt{L_1 C_1}}.$$ 

The quality factor is extremely high, with $Q \sim 3 \times 10^6$ (in vacuum) and about $Q = 1 \times 10^6$ (air). This is much higher than can be achieved with electrical circuit elements (inductors, capacitors, transmission lines, etc). This high $Q$ factor leads to good frequency stability (low phase noise).
MEMS Resonators

- The highest frequency, though, is limited by the thickness of the material. For $t \approx 15 \text{ } \mu\text{m}$, the frequency is about 200 MHz. MEMS resonators have been demonstrated up to $\sim \text{GHz}$ frequencies. MEMS resonators are an active research area.

- Integrated MEMS resonators are fabricated from polysilicon beams (forks), disks, and other mechanical structures. These resonators are electrostatically induced structures.

- We’ll come back to MEMS resonators in the second part of the lecture.
Some typical numbers for a fundamental mode resonator are $C_0 = 3 \text{ pF}$, $L_1 = 0.25 \text{ H}$, $C_1 = 40 \text{ fF}$, $R_1 = 50 \Omega$, and $f_0 = 1.6 \text{ MHz}$. Note that the values of $L_1$ and $C_1$ are modeling parameters and not physical inductance/capacitance. The value of $L$ is large in order to reflect the high quality factor.

The quality factor is given by

$$Q = \frac{\omega L_1}{R_1} = 50 \times 10^3 = \frac{1}{\omega R_1 C_1}$$
Recall that a series resonator has a phase shift from $-90^\circ$ to $+90^\circ$ as the impedance changes from capacitive to inductive form. The phase shift occurs rapidly for high $Q$ structures.

It's easy to show that the rate of change of phase is directly related to the $Q$ of the resonator

$$Q = \frac{\omega_s}{2} \left. \frac{d\phi}{d\omega} \right|_{\omega_0}$$

For high $Q$ structures, the phase shift is thus almost a “step” function unless we really zoom in to see the details.
In fact, it’s easy to show that the $\pm 45^\circ$ points are only a distance of $\omega_s/(2Q)$ apart.

$$\frac{\Delta \omega}{\omega_0} = \frac{1}{Q}$$

For $Q = 50 \times 10^3$, this phase change requires an only 20 ppm change in frequency.
Due to the external physical capacitor, there are two resonant modes between a series branch and the capacitor. In the series mode $\omega_s$, the $LCR$ is a low impedance (“short”). But beyond this frequency, the $LCR$ is an equivalent inductor that resonates with the external capacitance to produce a parallel resonant circuit at frequency $\omega_p > \omega_s$. 
In practice, any oscillator topology can employ a crystal as an effective inductor (between $\omega_s$ and $\omega_p$). The crystal can take on any appropriate value of $L_{\text{eff}}$ to resonate with the external capacitance.

Topologies that minimize the tank loading are desirable in order to minimize the XTAL de-Qing. The Pierce resonator is very popular for this reason.
Note that if the XTAL is removed from this circuit, the amplifier acts like a clock driver. This allows the flexibility of employing an external clock or providing an oscillator at the pins of the chip.
The thickness has tempco $t \sim 14 \text{ ppm/°C}$ leading to a variation in frequency with temperature. If we cut the XTAL in certain orientations ("AT-cut") so that the tempco of velocity cancels tempco of $t$, the overall tempco is minimized and a frequency stability as good as $f_0 \sim 0.6 \text{ ppm/°C}$ is possible.

Note that $1 \text{ sec/mo} = 0.4 \text{ ppm}$! Or this corresponds to only 0.4 Hz in 1 MHz.

This change in thickness for 0.4 ppm is only
\[ \delta t = 0.4 \times 10^{-6} \times t_0 = 0.4 \times 10^{-6} \times 10^{-3} \text{ m} = 4 \times 10^{-10}. \]
That’s about 2 atoms!

The smallest form factors available today’s AT-cut crystals are $2\times1.6 \text{ mm}^2$ in the frequency range of 24-54 MHz are available.
The typical temperature variation of the XTAL is shown. The variation is minimized at room temperature by design but can be as large as 15 ppm at the extreme ranges.

To minimize the temperature variation, the XTAL can be placed in an oven to form an Oven Compensated XTAL Oscillator, or OCXO. This requires about a cubic inch of volume but can result in extremely stable oscillator. OCXO $\sim 0.01$ ppm/°C.
In many applications an oven is not practical. A Temperature Compensated XTAL Oscillator, or TCXO, uses external capacitors to “pull” or “push” the resonant frequency. The external capacitors can be made with a varactor. This means that a control circuit must estimate the operating temperature, and then use a pre-programmed table to generate the right voltage to compensate for the XTAL shift. This scheme can achieve as low as $\text{TCXO} \sim 0.05 \text{ ppm/}^\circ\text{C}$.

Many inexpensive parts use a DCXO, or a digitally-compensated crystal oscillator, to eliminate the TCXO. Often a simple calibration procedure is used to set the XTAL frequency to within the desired range and a simple look-up table is used to adjust it.
Below series resonance, the equivalent circuit for the XTAL is a capacitor is easily derived.

The effective capacitance is given by

\[ C_{\text{eff}} = C_0 + \frac{C_x}{1 - \left(\frac{\omega}{\omega_s}\right)^2} \]
Past series resonance, the XTAL reactance is inductive

\[ jX_c = j\omega L_{eff} = \frac{1}{j\omega C_0} \left| j\omega L_x \left( 1 - \left( \frac{\omega_s}{\omega} \right)^2 \right) \right| \]

The XTAL displays \( L_{eff} \) from \( 0 \rightarrow \infty \) H in the range from \( \omega_s \rightarrow \omega_p \).
Thus for any \( C \), the XTAL will resonate somewhere in this range.
We can solve for the frequency range of \((\omega_s, \omega_p)\) using the following equation

\[
j \omega_p L_{\text{eff}} = \frac{1}{j \omega_p C_0} ||j \omega_p L_x \left(1 - \left(\frac{\omega_s}{\omega_p}\right)^2\right)\]

\[
\frac{\omega_p}{\omega_s} = \sqrt{1 + \frac{C_x}{C_0}}
\]

Example: \(C_x = 0.04 \text{ pF}\) and \(C_0 = 4 \text{ pF}\) Since \(C_0 \gg C_x\), the frequency range is very tight

\[
\frac{\omega_p}{\omega_s} = 1.005
\]
Consider now the series losses in the XTAL. Let
\[ X_1 = -\frac{1}{\omega C_1} \] and \[ X_2 = -\frac{1}{\omega C_2} \], and \[ jX_c = j\omega L_{\text{eff}} \].

Then the impedance \( Z'_L \) is given by

\[ Z'_L = \frac{jX_1 (R_x + jX_c + jX_2)}{R_x + (jX_c + jX_1 + jX_2)} \equiv 0 \text{ at resonance} \]
It follows therefore that $jX_c + jX_2 = -jX_1$ at resonance and so

$$Z'_L = \frac{jX_1(R_x - jX_1)}{R_x} = \frac{(X_1 + jR_x)X_1}{R_x}$$

Since the $Q$ is extremely high, it’s reasonable to assume that $X_c \gg R_x$ and thus $X_1 + X_2 \gg R_x$, and if these reactances are on the same order of magnitude, then $X_1 \gg R_x$. Then

$$Z'_L \approx \frac{X_1^2}{R_x}$$

This is the XTAL loss reflected to the output of the oscillator.
Losses at Overtones

- Since $X_1$ gets smaller for higher $\omega$, the shunt loss reflected to the output from the overtones gets smaller (more loading).
- The loop gain is therefore lower at the overtones compared to the fundamental in a Pierce oscillator.
- For a good design, we ensure that $A_\ell < 1$ for all overtones so that only the dominate mode oscillates.
The design of a XTAL oscillator is very similar to a normal oscillator. Use the XTAL instead of an inductor and reflect all losses to the output.

\[ A_\ell = g_m R_L \frac{C_1}{C_2} \]

\[ R_L = R'_L || R_B || r_o || \cdots \]

For the steady-state, simply use the fact that \( G_m R_L \frac{C_1}{C_2} = 1 \), or \( G_m/g_m = 1/A_\ell \).
For a second order system, the poles are placed on the circle of radius $\omega_0$. Since the envelope of a small perturbation grows like

$$v_0(t) = Ke^{\sigma_1 t} \cos \omega_0 t$$

where $\sigma_1 = 1/\tau$ and $\tau = \frac{Q}{\omega_0} \frac{2}{A_\ell - 1}$.

For example if $A_\ell = 3$, $\tau = \frac{Q}{\omega_0}$. That means that if $Q \sim 10^6$, about a million cycles of simulation are necessary for the amplitude of oscillation to grow by a factor of $e \approx 2.71$!
Since this can result in a very time consuming transient (TRAN) simulation in SPICE, you can artificially de-Q the tank to a value of $Q \sim 10 - 100$. Use the same value of $R_x$ but adjust the values of $C_x$ and $L_x$ to give the right $\omega_0$ but low $Q$.

Alternatively, if PSS or harmonic balance (HB) are employed, the steady-state solution is found directly avoiding the start-up transient.

Transient assisted HB and other techniques are described in the ADS documentation.
Note that the LCR tank is a low $Q(3-20)$ tuned to the approximate desired fundamental frequency of the XTAL (or overtone).

The actual frequency selectivity comes from the XTAL, not the LCR circuit. The LCR loaded $Q$ at resonance is given by the reflected losses at the tank

$$R_T' = R_T \| n^2 (R_x + R_B')$$

$$R_B' = R_B \| R_i|_{DP}$$
At resonance, the loop gain is given by

\[ A_\ell = G_m R'_T \frac{C_1}{C_1 + C_2} \frac{R'_B}{R'_B + R_x} \]

The last term is the resistive divider at the base of Q1 formed by the XTAL and the biasing resistor.

In general, the loop gain is given by

\[ A_\ell = G_m Z_T(j\omega) \frac{C_1}{C_1 + C_2} \frac{R'_B}{R'_B + Z_x(j\omega)} \]

The first term \( A_{\ell,1} \) is not very frequency selective due to the low \( Q \) tank. But \( A_{\ell,2} \) changes rapidly with frequency.
In this case the low $Q$ tank selects the fundamental mode and the loop gain at all overtones is less than unity.
In this case the low $Q$ tank selects a $3\omega_0$ overtone mode and the loop gain at all other overtones is less than unity. The loop gain at the fundamental is likewise less than unity.
For communication systems we need precise frequency references, stable over temperature and process, with low phase noise. We also need to generate different frequencies “quickly” to tune to different channels.

XTAL’s are excellent references but they are at lower frequencies (say below 200 MHz) and fixed in frequency. How do we synthesize an RF and variable version of the XTAL?
This is a “phase locked loop” frequency synthesizer. The stable XTAL is used as a reference. The output of a VCO is phase locked to this stable reference by dividing the VCO frequency to the same frequency as the reference.

The phase detector detects the phase difference and generates an error signal. The loop filter thus will force phase equality if the feedback loop is stable.