Mixers: An Overview

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The Mixer is a critical component in communication circuits. It translates information content to a new frequency.

On the transmitter, baseband data is up-converted to an RF carrier (shown above).

In a receiver, the same information is ideally down-converted to baseband.
**Mixers Specifications**

- **Conversion Gain**: Ratio of voltage (power) at output frequency to input voltage (power) at input frequency
  - Downconversion: RF power in, IF power out
  - Up-conversion: IF power in, RF power out
- **Noise Figure**
  - DSB versus SSB
- **Linearity**
- **Image Rejection, Spurious Rejection**
- **LO Feedthrough**
  - Input
  - Output
- **RF Feedthrough**
We know that any non-linear circuit acts like a mixer.
Squarer Example

Product component:

\[ y = A^2 \cos^2 \omega_1 t + B^2 \cos^2 \omega_2 t + 2AB \cos \omega_1 t \cos \omega_2 t \]

What we would prefer:

\[ v_{RF} = v_{RF} \cos \omega_1 t \]
\[ v_{LO} = v_{RF} \cos \omega_1 t \]
\[ v_{IF} = 2v_{LO} \cdot v_{RF} \cos (\omega_1 \pm \omega_2) t \]

A true quadrant multiplier with good dynamic range is difficult to fabricate.
LTV Mixer

\[ f_1 + f_2 \rightarrow \text{LTI} \rightarrow f_1 + f_2 \quad \text{No new frequencies} \]

\[ f_1 + f_2 \rightarrow \text{LTV} \rightarrow \text{New tones in output} \]

- No new frequencies for a Linear Time Invariant (LTI) system
- But a Linear Time-Varying (LTV) “Mixer” can act like a multiplier
- Example: Suppose the resistance of an element is modulated periodically

\[ v_o = i_{in} \cdot R(t) \]
\[ = I_o \cos(\omega_{RF} t) \cdot R_o \cos(\omega_{LO} t) \]
\[ = \frac{I_o R_o}{2} \{ \cos(\omega_{RF} + \omega_{LO}) t + \cos(\omega_{RF} - \omega_{LO}) t \} \]
Periodically Time Varying Systems

- In general, any periodically time varying system can achieve frequency translation

\[ v(t) = p(t) v_i(t) \]

\[ p(t + T) = p(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 n t} v_i(t) \]

\[ c_n = \frac{1}{T} \int_{0}^{T} p(t) e^{-j\omega_0 n t} dt \]

\[ v_i(t) = A(t) \cos \omega_1 t = A(t) \left( \frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2} \right) \]

\[ v_o(t) = A(t) \sum_{-\infty}^{\infty} c_n \frac{e^{j(\omega_0 n t + \omega_1 t)} + e^{j(\omega_0 n t - \omega_1 t)}}{2} \]

- Consider \( n=1 \) plus \( n=-1 \)
Desired Mixing Product

\[ c_1 = c_{-1} \]

\[ v_o(t) = \frac{c_1}{2} e^{j(\omega_o t - \omega_1 t)} + \frac{c_{-1}}{2} e^{-j(\omega_o t + \omega_1 t)} \]

\[ = c_1 \cos(\omega_o t - \omega_1 t) \]

- Output contains desired signal (plus a lot of other signals). Must pre-filter the undesired signals (such as the image band).
Convolution in Frequency

- Ideal multiplier mixer:

\[ y(t) = p(t) \times x(t) \]

\[ Y(f) = X(f) \ast P(f) \]

\[ P(f) = \sum_{-\infty}^{\infty} c_n \delta(f - n f_{LO}) \]

\[ Y(f) = \int_{-\infty}^{\infty} \sum_{-\infty}^{\infty} c_n \delta(\sigma - n f_{LO}) X(f - \sigma) \, d\sigma \]

\[ = \sum_{-\infty}^{\infty} c_n \left( \int_{-\infty}^{\infty} \delta(\sigma - n f_{LO}) X(f - \sigma) \, d\sigma \right) \]

\[ = \sum_{-\infty}^{\infty} c_n X(f - n f_{LO}) \]
Input spectrum is translated into multiple points at the output at multiples of the LO frequency.
Filtering is required to reject the undesired harmonics.
Output spectrum is translated from multiple sidebands (both sides – image issue) into the same output.

This is the origin of the lack of image and harmonic rejection in a basic mixer. We have already seen image reject architectures. Later we’ll study harmonic rejection mixers.
Balanced Mixer Topologies
Balanced Mixer

- An unbalanced mixer has a transfer function:
\[ y(t) = x(t) \times s(t) = (1 + A(t) \cos(\omega_{RF}t)) \times \begin{cases} 0 & LO < 0 \\ 1 & LO > 0 \end{cases} \]

- which contains both RF, LO, and IF

- For a single balanced mixer, the LO signal is “balanced” (bipolar) so we have
\[ y(t) = x(t) \times s(t) = (1 + A(t) \cos(\omega_{RF}t)) \times \begin{cases} -1 & LO < 0 \\ +1 & LO > 0 \end{cases} \]

- As a result, the output contains the LO but no RF component

- For a double balanced mixer, the LO and RF are balanced so there is no LO or RF leakage
\[ y(t) = x(t) \times s(t) = A(t) \cos(\omega_{RF}t) \times \begin{cases} -1 & LO < 0 \\ +1 & LO > 0 \end{cases} \]
\[ I_{o1} = I_1 - I_2 = F(V_{LO}(t), I_B + i_s) \]

- Assume \( i_s \) is small relative to \( I_B \) and perform Taylor series expansion.

\[ I_{o1} \approx F(V_{LO}(t), I_B) + \frac{\partial F}{\partial I_B} (V_{LO}(t), I_B) \cdot i_s + ... \]

\[ I_{o1} = P_o(t) + P_1(t) \cdot i_s \]
\[
\frac{i_1}{i_s} = \frac{1}{g_{m1}} + \frac{1}{g_{m2}} \\
\frac{i_2}{i_s} = \frac{1}{g_{m1}} + \frac{1}{g_{m2}}
\]

\[
p_1(t) = \frac{g_{m1}(t) - g_{m2}(t)}{g_{m1}(t) + g_{m2}(t)} \left( = \frac{i_1 - i_2}{i_s} \right)
\]

- Note that with good device matching: \( p_1(t) = -p_1(t + \frac{T_o}{2}) \)
- Expand \( p_1(t) \) into a Fourier series:

\[
p_{1,2k} = \frac{1}{T_{LO}} \int_0^{T_{LO}} p_1(t) e^{-j2\pi2kt/T_{LO}} dt = \int_0^{T_{LO}/2} + \int_{T_{LO}/2}^{T_{LO}} = 0
\]

- Only odd coefficients of \( p_{1,n} \) non-zero
Assume LO signal strong so that current (RF) is alternatively sent to either M2 or M3. This is equivalent to multiplying $i_{RF}$ by $\pm 1$.

$$v_{IF} \approx \text{sign} (V_{LO}) g_m R_L v_{RF} = g(t) g_m R_L v_{RF}$$

$g(t)$ periodic waveform with period $= T_{LO}$
\[ g(t) = \text{square wave} = \frac{4}{\pi} (\cos \omega_{LO} t - \cos 3\omega_{LO} t + \ldots) \]

- Let \( v_{RF} = A \cos \omega_{RF} t \)
- Gain:
  \[ A_v = \frac{\tilde{v}_{IF}}{A} = \frac{1}{2 \pi} g_m R_L = \frac{2}{\pi} g_m R_L \]
- LO-RF isolation good, but LO signal appears in output (just a diff pair amp).
- Strong LO might desensitize (limit) IF stage (even after filtering).
Double Balanced Mixer

LO signal is rejected up to matching constraints
Differential output removes even order non-linearities
Linearity is improved: Half of signal is processed by each side
Noise higher than single balanced mixer since no cancellation occurs
Mixer Design Examples
Role of input stage is to provide $V$-to-$I$ conversion. Degeneration helps to improve linearity. Inductive degeneration is preferred as there is no loss in headroom and it provides input matching.

An LO trap also minimizes swing of LO at output which may cause premature compression.
Common Gate Input Stage

\[ V_{bias} \]

\[ I_{out} \]

\[ V_{in} \]

\[ V_{mirr} \]
The LNA output is often single-ended. A good balanced RF signal is required to minimize the feedthrough to the output. LC bridge circuits can be used, but the bandwidth is limited. A transformer is a good choice for this, but bulky and bandwidth is still limited.

A broadband single-ended to differential conversion stage can be used to generate highly balanced signals over very wide bandwidths. $G_m$ stage is Class AB.
Set $I_Z$ to bias for match: $R_{IN} = 2V_t/I_Z$.

Using “trans-linear concept” to derive currents by defining $\lambda = I_{RF}/2I_Z$

\[
\begin{align*}
I_Z^2 &= I_1 I_2 = I_1 (I_1 + I_{RF}) \\
I_{1,2} &= I_Z \left( \sqrt{\lambda^2 + 1} \mp \lambda \right) \\
I_1 - I_3 &= 2\lambda I_Z = I_{RF} \\
R_{IN}(\lambda) &= \frac{V_t}{2I_Z} \frac{1}{\sqrt{\lambda^2 + 1}}
\end{align*}
\]
For broadband applications, an active balun is a good solution but impacts linearity. A fully passive balun can be designed with good bandwidth and balance, resulting in very good overall linearity. Watch out for common mode coupling in balun.
Large currents are good for the \( g_m \) stage (noise, conversion gain), but require large devices in the switching core \( \Rightarrow \) hard to switch due to capacitance and also requires a large LO (large \( V_{gs} - V_t \))

A current source can be used to feed the \( G_m \) stage with extra current.
Current Re-Use Gm Stage

- **Note that the PMOS currents are out of phase with the NMOS, and hence the inverted polarity for the LO is used to compensate.**

- **CMOS technology has fast PMOS devices which can be used to increase the effective $G_m$ of the transconductance stage without increasing the current by stacking.**
Note that for weak signals, the second-order non-linearity will mix the LO and RF signals.

We prefer to apply a strong LO to periodically modulate the transconductance $G_m(t)$ of the transistor.
Unfold the quad, we get a ring. They are the same!

Notice that a mixer is just a circuit that flips the sign of the transfer function from ± every cycle of the LO. We can either steer currents or voltages.

The switches can be actively biased and switched with an LO of approximately a few $V_{gs} - V_T$ (or several times $kT/q$ for BJT), or they can be biased passively and fully switched on and off.
Passive Mixers ($V$)

- Devices act as switches and just rewire the circuit so that the plus/minus voltage is connected to the output with ±1 polarity.
- Very linear but requires a high LO swing and larger LO power.

![Circuit diagram](image)

\[ \text{LO} \quad \text{RF} \quad \text{RF} \quad \text{LO} \quad \text{LO} \quad \text{IF} \quad R_{IF} \]

\[ +RF \quad R_{s/2} \quad +RF \quad R_{s/2} \quad -RF \quad -RF \]

\[ L_1 \quad C_3 \quad L_3 \quad \text{LO} \quad \text{LO} \quad \text{IF} \quad \text{LO} \]

\[ L_2 \]

Niknejad Advanced IC's for Comm
Commutate currents using passive switches. Very linear (assuming current does not have distortion) but requires higher LO drive.

Good practice to DC bias switches at desired operating point and AC coupling the $G_m$ stage.

Since we’re processing a current, we use a TIA to present a low impedance and convert $I$-to-$V$. 
Sub-Sampling Mixers

Note that sampling is equivalent to multiplication by an impulse train (spacing of $T_{LO}$), which in the frequency domain is the convolution of another impulse train (spacing of $1/T_{LO}$).

This means that we can sample or even sub-sample the signal. The problem is that noise from all harmonics of the LO folds to common IF. Always true, but especially problematic for sub-sampling.
Gain programmed using current through M16 (set by resistance of triode region devices)
Common mode feedback to set output point
Cascode improves isolation (LO to RF)
Power Spectral Density
Average response of LTI system:

\[ y_1(t) = H_1[x(t)] = \int_{-\infty}^{\infty} h_1(t) x(t - \tau) d\tau \]

\[ \overline{y_1(t)} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y_1(t) dt \]

\[ = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left( \int_{-\infty}^{\infty} h_1(\tau) x(t - \tau) d\tau \right) dt \]

\[ = \int_{-\infty}^{\infty} \left( \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t - \tau) dt \right) h_1(\tau) dt \]
Average Value Property

\[ \overline{y_1(t)} = \overline{x(t)} \int_{-\infty}^{\infty} h_1(t) \, dt \]

\[ H_1(j\omega) = \int_{-\infty}^{\infty} h_1(t) e^{-j\omega t} \, dt \]

\[ \overline{y_1(t)} = \overline{x(t)} H_1(0) \]
\[
\bar{y_1^2}(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left( \int_{-\infty}^{\infty} h_1(\tau_1) x(t - \tau_1) d\tau_1 \right) \left( \int_{-\infty}^{\infty} h_1(\tau_2) x(t - \tau_2) d\tau_2 \right) dt
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau_1) h_1(\tau_2) \left( \lim_{T \to \infty} \frac{1}{2\pi} \int_{-T}^{T} x(t - \tau_1) x(t - \tau_2) dt \right) d\tau_1 d\tau_2
\]

- Recall the definition for the autocorrelation function

\[
\varphi_{xx}(t) = \overline{x(t) x(t + \tau)}
\]

\[
= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) x(t + \tau) dt
\]
Autocorrelation Function

\[
\bar{y_1}^2(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau_1) h_2(\tau_2) \phi_{xx}(\tau_1 - \tau_2) d\tau_1 d\tau_2
\]

\[
\phi_{xx}(j\omega) = \int_{-\infty}^{\infty} \phi_{xx}(\tau) e^{-j\omega\tau} d\tau
\]

\[
\phi_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{xx}(j\omega) e^{j\omega\tau} d\omega
\]

- \(\phi_{xx}(j\omega)\) is a real and even function of \(\omega\) since \(\phi_{xx}(t)\) is a real and even function of \(t\)
Autocorrelation Function (2)

\[
\overline{y_1^2(t)} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau_1) h_1(\tau_2) \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{xx}(j\omega) e^{j\omega(\tau_1 - \tau_2)} d\omega d\tau_1 d\tau_2
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{xx}(j\omega) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau_1) h_1(\tau_2) e^{j\omega(\tau_1 - \tau_2)} d\tau_1 d\tau_2
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{xx}(j\omega) \left( \int_{-\infty}^{\infty} h_1(\tau_1) e^{+j\omega \tau_1} d\tau_1 \right) \left( \int_{-\infty}^{\infty} h_1(\tau_2) e^{-j\omega \tau_2} d\tau_2 \right) d\omega
\]

\[
\overline{y_1^2(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{xx}(j\omega) H_1(j\omega) H_1^*(j\omega) d\omega
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{xx}(j\omega) |H_1(j\omega)|^2 d\omega
\]
Consider $x(t)$ as a voltage waveform with total average power $x^2(t)$. Let's measure the power in $x(t)$ in the band $0 < f < f_1$.

The average power in the frequency range $0 < f < f_1$ is now

$$
y_1^2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{xx}(j\omega) |H_1(j\omega)|^2 d\omega
$$

$$
= \frac{1}{2\pi} \int_{-\omega_1}^{\omega_1} \varphi_{xx}(j\omega) d\omega = \int_{-f_1}^{f_1} \varphi_{xx}(j2\pi f) df
$$
Average Power in $X(t)$ (2)

$$\begin{align*}
&= 2 \int_{0}^{f_1} \varphi_{xx} (j2\pi f) \, df \\
&= 2 \int_{f_1}^{f_2} \varphi_{xx} (j2\pi f) \, df
\end{align*}$$

- Generalize: To measure the power in any frequency range, apply an ideal bandpass filter with passband $f_1 < f < f_2$

$$\overline{y_1^2(t)} = 2 \int_{f_1}^{f_2} \varphi_{xx} (j2\pi f) \, df$$

- The interpretation of $\varphi_{xx}$ as the the power spectral density (PSD) is clear.
A spectrum analyzer measures the PSD of a signal

“Poor Man’s” spectrum analyzer:

- VCO
- Sweep generation
- Linear wide tuning range
- Wide dynamic range mixer
- Sharp filter
- CRT