Wireless Receiver Architectures

Prof. Ali M. Niknejad and Dr. Osama Shana’a

U.C. Berkeley
Copyright © 2014 by Ali M. Niknejad
Outline

- Review of key receiver specs
- Complex baseband equivalent of a bandpass signal
- Superheterodyne
- Image Rejection Architectures
- Direct-conversion
- References
Review of Receiver Specifications
Receiver Specifications

- **Sensitivity**, the weakest signal that can be detected by a receiver. Set by amount of *Amplification* and *Noise*
- **Selectivity**, or the ability to select one frequency band in the presence of interfering signals in nearby frequency bands. Determined by the amount of *Filtering* and the receiver *Linearity*
- **Dynamic Range**, or the linear range of a receiver signal that can be processed with minimal impact on the detection bit error rate (BER), related to the *Linearity* and *Noise* in the receiver. Greatly improved with *Variable Gain Control* (or *Programmable Gain Control*)
A detector works well with a fairly strong signal. For instance, if the input referred noise/offset/LSB is 10’s mV, the input signal should be 10X or more larger.

The weakest signals can be -100 dBm or lower, so a gain of $> 90$dB may be needed. What happens when the input signal is 0 dBm???

In gaining up the signal, we have to keep the noise and distortion small relative to the signal power in order to meet the required SNDR.

SDR Concept (minimize RF gain): ADC spec of $> 100$dB of dynamic range ($> 17$bit) with a sampling frequency of $2f_{RF}$. 
Since the received power can vary greatly in dynamic range from very weak levels (-110 dBm) to fairly strong signals (0 dBm), the receiver should ideally have variable gain of $\sim 0$-100 dB.

Without variable gain, the dynamic range of a receiver is limited since the detector or ADC may have a limited range. For an ADC, it’s roughly 6 dB/bit, and the power consumption grows exponentially with the number of bits.

Variable gain at RF is difficult. But even if implemented, the sensitivity would be limited by the presence of interfering signals.
As the carrier frequency and the information signal are at very disparate frequencies (say 1 GHz versus 1 MHz), we require modulation and demodulation.

Also, we prefer to work at lower frequencies to save power. We would like to frequency translate our signal to “baseband” and perform filtering/gain rather than at RF. This means we should “mix” the signal as soon as possible.

We shall see that mixers are prone to frequency translate many different frequencies to the same “IF”, and so they are relatively noisy and require image rejection (NF $\sim 10$ dB). We must precede the mixers with a low noise amplifier (LNA) to overcome this noise.
Imagine trying to receive a signal at a power of -100 dBm in the presence of an inband “jammer” or interference signal with power -40 dBm.

We would like to set the gain at 100 dB, but this would severely compress the receiver due to the jammer.

We must therefore apply a sharp filter to remove the jamming signal before we apply all the gain.

If these jammers (blockers/interferers) are not attenuated, they tend to reduce the gain of the signal ($P_{-1dB}$), increase the noise figure of the receiver (through mixing noise in other bands to the same IF, especially phase noise), and produce intermodulation products that fall in band and reduce the sensitivity of a receiver.
Filtering in Receivers

Niknejad
Advanced IC's for Comm
• High gain required to meet sensitivity (∼ 70dB)
• DC offset cancellation is a must (due to mismatches)
• Gain control to accommodate strong blockers
• High linearity filtering stages to knock down blockers
• Image rejection and harmonic rejection (cannot be filtered easily)
• Tunable baseband IF corner frequency (200 kHz  100 MHz) for different standards
Complex Baseband Representation
Any passband waveform can be written in the following form:

\[ s_p(t) = a(t) \cos [\omega_c t + \theta(t)] \]

\[ s_p(t) = a(t) \cos \omega_c t \cos \theta(t) - a(t) \sin \omega_c t \sin \theta(t) \]

\[ s_p(t) = \sqrt{2} s_c(t) \cos \omega_c t - \sqrt{2} s_s(t) \sin \omega_c t \]

\[ s_c(t) \triangleq a(t) \cos \theta(t) = I(t) \]

\[ s_s(t) \triangleq a(t) \sin \theta(t) = Q(t) \]

\[ a(t) = |s(t)| = \sqrt{s_c^2(t) + s_s^2(t)} \]

\[ \theta(t) = \tan^{-1} \frac{s_s(t)}{s_c(t)} \]
We define the complex baseband signal and show that all operations at passband have a simple equivalent at complex baseband:

\[ s(t) = s_c(t) + js_s(t) = I(t) + jQ(t) \]

\[ s_p(t) = \text{Re} \left\{ \sqrt{2} s(t) e^{j\omega_c t} \right\} \]

\[ ||s||^2 = ||s_p||^2 \]
An important relationship is the orthogonality between the modulated I and Q signals: $< x_c, x_s > = < X_c, X_s > = 0$

This can be proved as follows (Parseval’s Relation):

$$< X_c, X_s > = \int_{-\infty}^{\infty} X_c(f)X_s^*(f)df$$

$$x_c(t) = \sqrt{2} s_c(t) \cos \omega_c t \quad x_s(t) = \sqrt{2} s_s(t) \sin \omega_c t$$

$$x_c(t) = \frac{1}{\sqrt{2}} (s_c(t)e^{j\omega_c t} + s_c(t)e^{-j\omega_c t})$$

$$X_c(f) = \frac{1}{\sqrt{2}} (S_c(f - f_c) + S_c(f + f_c))$$

$$X_s(f) = \frac{1}{\sqrt{2}j} (S_s(f - f_c) - S_s(f + f_c))$$
Orthogonality

\[ < X_c, X_s > = \]

\[
\frac{1}{2j} \int_{-\infty}^{\infty} \left( (S_c(f - f_c) + S_c(f + f_c)) \times (S_s^*(f - f_c) - S_s^*(f + f_c)) \right) df
\]

- In the above integral, if the carrier frequency is larger than the signal bandwidth, then the frequency shifted signals do not overlap

\[
S_c(f - f_c)S_s^*(f + f_c) \equiv 0
\]

\[
S_c(f + f_c)S_s^*(f - f_c) \equiv 0
\]

\[ < X_c, X_s > = \frac{1}{2j} \left[ \int_{-\infty}^{\infty} S_c(f - f_c)S_s^*(f - f_c) df - \int_{-\infty}^{\infty} S_c(f + f_c)S_s^*(f + f_c) df \right] \]

\[ < X_c, X_s > = \frac{1}{2j} \left[ \int_{-\infty}^{\infty} S_c(f)S_s^*(f) df - \int_{-\infty}^{\infty} S_c(f)S_s^*(f) df \right] = 0 \]
Due to this orthogonality, we can double the bandwidth of our signal by modulating the I and Q independently. Also, we have

\[ <u_p, v_p> = <u_c, v_c> + <u_s, v_s> = \text{Re}[<u, v>] \]

Since the passband signal is real, it has a conjugate symmetric spectrum about the origin. Let’s define the positive portion as follows:

\[ S_p^+(f) = S_p(f)u(f) \]

Then the spectrum of the passband and baseband complex signal are related by:

\[ S(f) = \sqrt{2}S_p^+(f + f_c) \quad S_p(f) = \frac{S(f-f_c) + S^*(-f-f_c)}{\sqrt{2}} \]
Proof:

\[ \nu(t) = \sqrt{2} s(t)e^{j\omega_c t} \]

\[ V(f) = \sqrt{2} S(f - f_c) \]

\[ S_p(t) = \text{Re}(\nu(t)) = \frac{\nu(t) + \nu(t)^*}{2} \]

\[ S_p(f) = \frac{V(f) + V^*(-f)}{2} = \frac{S(f - f_c) + S^*(-f - f_c)}{\sqrt{2}} \]
The Image Problem

\[ m_r(t) \cos(\omega_{LO}+\omega_{IF})t \times \cos(\omega_{LO})t = \frac{1}{2} m_r(t) (\cos(2\omega_{LO} + \omega_{IF})t + \cos(\omega_{IF})t) \]

\[ m_i(t) \cos(\omega_{LO}-\omega_{IF})t \times \cos(\omega_{LO})t = \frac{1}{2} m_i(t) (\cos(2\omega_{LO} - \omega_{IF})t + \cos(\omega_{IF})t) \]

- After low-pass filtering the mixer output, the IF is given by

\[ IF_{output} = \frac{1}{2} (m_i(t) + m_r(t)) \cos(\omega_{IF})t \]
Complex modulation shifts in only one direction . . . real modulation shifts up and down
Superheterodyne Architecture
The choice of the IF frequency dictated by:

- If the IF is set too low, then we require a very high-Q image reject filter, which introduces more loss and therefore higher noise figure in the receiver (not to mention cost).
- If the IF is set too high, then subsequent stages consume more power (VGA and filters)
- Typical IF frequency is 100-200 MHz.
Two separate VCOs and synthesizers are used. The IF LO is fixed, while the RF LO is variable to down-convert the desired channel to the passband of the IF filter (SAW). This typically results in a 3-4 chip solution with many off-chip components.

$LO_1$ should never be made close to an integer multiple of $LO_2$ for any channel. The $N$th harmonic of the fixed $LO_2$ could leak into the RF mixer and cause unwanted mixing.
The 1/2 IF Problem

Assume that there is a blocker half-way between the LO and the desired channel. Due to second-order non-linearity in the RF front-end:

\[
\left[ m_{\text{blocker}}(t) \cos(\omega_{LO} + \frac{1}{2} \omega_{IF} t) \right]^2 = m_{\text{blocker}}^2(t) \frac{1 + \cos(2\omega_{LO} + \omega_{IF} t)}{2}
\]

If the LO has a second-order component, then this signal will fold right on top of the desired signal at IF:

\[
\left[ m_{\text{blocker}}^2(t) \cos(2\omega_{LO} + \omega_{IF} t) \right] \cos(2\omega_{LO} t) = m_{\text{blocker}}^2(t) \cos(\omega_{IF} t) + \cdots
\]

Note: Bandwidth expansion of blocker due to squaring operation.
If the IF stage has strong second-order non-linearity, then the half-IF problem occurs through this mechanism:

\[
2 \left[ m_{blocker}(t) \cos(\frac{1}{2}\omega_{IF}t) \right]^2 = m_{blocker}^2(t) + m_{blocker}^2(t) \cos(\omega_{IF}t)
\]

This highlights the importance of frequency planning. One should select the IF by making sure that there is no strong half-IF blocker. If one exists, then the second-order non-linearity must be carefully managed.
When we down-convert twice using a mixer, we have to make sure that the image in both the image bands is suppressed. The secondary mixer has an image that may fall in a band close to our desired signal, making the second image rejection difficult.

\[
IM_2 = IF_1 - 2IF_2 = IF_1 - 2(IF_1 - LO_2) = -IF_1 + 2LO_2
\]
\[
IM'_2 = IM_2 + LO_1 = LO_1 + 2LO_2 - IF_1 = LO_1 + 2LO_2 - (RF - LO_1) = 2(LO_1 + LO_2) - RF
\]

Note \( LO_1 + LO_2 = RF - IF_2 \)

\[
IM'_2 = 2RF - 2IF_2 - RF = RF - 2IF_2
\]
Disadvantages:

- Requires bulky off-chip SAW filters
- As before, two synthesizers are required
- Typically a three chip solution (RF, IF, and Synth)

Advantages:

- Robust. The clear choice for extremely high sensitivity radios
- High dynamic range SAW filter reduces/relaxes burden on active circuits. This makes it much easier to design the active circuitry.
- By the same token, the power consumption is lower
Image Reject Architectures
A complex mixer is derived by simple substitution.

Note that a complex exponential only introduces a frequency shift in one direction (no image rejection problems).
Image suppression by proper phase shifting.

\[ RF = m_r(t) \cos(\omega_{LO} + \omega_{IF})t + m_i(t) \cos(\omega_{LO} - \omega_{IF})t \]

\[ A = RF \times \cos(\omega_{LO}t) = \frac{1}{2} m_r(t) (\cos(2\omega_{LO} + \omega_{IF})t + \cos(\omega_{IF})t) + \frac{1}{2} m_i(t) (\cos(2\omega_{LO} - \omega_{IF})t + \cos(\omega_{IF})t) \]

\[ B = RF \times \sin(\omega_{LO}t) = \frac{1}{2} m_r(t) (\sin(2\omega_{LO} + \omega_{IF})t - \sin(\omega_{IF})t) + \frac{1}{2} m_i(t) (\sin(2\omega_{LO} - \omega_{IF})t + \sin(\omega_{IF})t) \]

\[ C = \frac{1}{2} m_r(t) (-\cos(2\omega_{LO} + \omega_{IF})t + \cos(\omega_{IF})t) + \frac{1}{2} m_i(t) (-\cos(2\omega_{LO} - \omega_{IF})t - \cos(\omega_{IF})t) \]

\[ IF^+ = A + C = m_r(t) \cos(\omega_{IF}t) \]

\[ IF^- = A - C = m_i(t) \cos(\omega_{IF}t) \]
Since the sine treats positive/negative frequencies differently (above/below LO), we can exploit this behavior.

- A $90^\circ$ phase shift is needed to eliminate the image.
- $90^\circ$ phase shift equivalent to multiply by $-j\text{sign}(f)$.
Hilbert Implementation

- **Advantages:**
  - Remove the external image-reject SAW filter
  - Better integration

- Requires extremely good matching of components (paths gain/phase). Without trimming/calibration, only $\sim 40$dB image rejection is possible. Many applications require $60$dB or more.

- Power hungry (more mixers and higher cap loading).

- Note: A real implementation uses $45^\circ/135^\circ$ phase shifters for better matching/tracking.
Gain/Phase Imbalance

\[ A = RF \times (1 + \alpha) \cos(\omega_{LO}t + \frac{\phi}{2}) = \frac{1}{2} m_r(t)(1 + \alpha) \left( \cos(2\omega_{LO}t + \omega_{IF}t + \frac{\phi}{2}) + \cos(\omega_{IF}t - \frac{\phi}{2}) \right) + \frac{1}{2} m_i(t)(1 + \alpha) \left( \cos(2\omega_{LO}t - \omega_{IF}t + \frac{\phi}{2}) + \cos(\omega_{IF}t + \frac{\phi}{2}) \right) \]

\[ B = RF \times (1 - \alpha) \sin(\omega_{LO}t - \frac{\phi}{2}) = \frac{1}{2} m_r(t)(1 - \alpha) \left( \sin(2\omega_{LO}t + \omega_{IF}t - \frac{\phi}{2}) - \sin(\omega_{IF}t - \frac{\phi}{2}) \right) + \frac{1}{2} m_i(t)(1 - \alpha) \left( \sin(2\omega_{LO}t - \omega_{IF}t - \frac{\phi}{2}) + \sin(\omega_{IF}t - \frac{\phi}{2}) \right) \]

\[ C = \frac{1}{2} m_r(t)(1 - \alpha) \left( -\cos(2\omega_{LO}t + \omega_{IF}t - \frac{\phi}{2}) + \cos(\omega_{IF}t - \frac{\phi}{2}) \right) + \frac{1}{2} m_i(t)(1 - \alpha) \left( -\cos(2\omega_{LO}t - \omega_{IF}t - \frac{\phi}{2}) - \cos(\omega_{IF}t - \frac{\phi}{2}) \right) \]

\[ IF = A + C = \frac{m_r(t)}{2} \left( (1 + \alpha) \cos(\omega_{IF}t - \frac{\phi}{2}) + (1 - \alpha) \cos(\omega_{IF}t + \frac{\phi}{2}) \right) + \frac{m_i(t)}{2} \left( (1 + \alpha) \cos(\omega_{IF}t + \frac{\phi}{2}) - (1 - \alpha) \cos(\omega_{IF}t - \frac{\phi}{2}) \right) \]

\[ IF = m_r(t) \left[ \cos(\omega_{IF}t) \cos(\frac{\phi}{2}) - \alpha \sin(\omega_{IF}t) \sin(\frac{\phi}{2}) \right] + m_i(t) \left[ \alpha \cos(\omega_{IF}t) \cos(\frac{\phi}{2}) - \sin(\omega_{IF}t) \sin(\frac{\phi}{2}) \right] \]
Image-Reject Ratio

\[ IR(dB) = 10 \log \left( \frac{\cos \frac{\phi}{2} - \alpha \sin \frac{\phi}{2}}{\alpha \cos \frac{\phi}{2} + \sin \frac{\phi}{2}} \right)^2 \]

\[ IR \approx \frac{\alpha^2 + \phi^2}{4} \]

- Level of image rejection depends on amplitude/phase mismatch
- Typical op-chip values of 30-40 dB achieved (< 5°, < 0.6dB)
RF = m_r(t) \cos(\omega_{LO} + \omega_{IF})t + m_i(t) \cos(\omega_{LO} - \omega_{IF})t

RF' = -m_r(t) \sin(\omega_{LO} + \omega_{IF})t - m_i(t) \sin(\omega_{LO} - \omega_{IF})t

A = RF \times \cos(\omega_{LO} t) = \frac{1}{2} m_r(t) (\cos(2\omega_{LO} + \omega_{IF})t + \cos(\omega_{IF})t) + \frac{1}{2} m_i(t) (\cos(2\omega_{LO} - \omega_{IF})t + \cos(\omega_{IF})t)

A'_{LPF} = -m_r(t) \sin(\omega_{IF} t) - m_i(t) \sin(\omega_{IF} t)

B = RF' \times \cos(\omega_{LO} t) = \frac{1}{2} m_r(t) (-\sin(2\omega_{LO} + \omega_{IF})t + \sin(\omega_{IF})t) + \frac{1}{2} m_i(t) (-\sin(2\omega_{LO} - \omega_{IF})t - \sin(\omega_{IF})t)

IF^+ = B - A' = m_r(t) \sin(\omega_{IF} t)

- This requires a 90° phase shift across the band. It’s much easier to shift the phase of a single frequency (LO). Even though the LO is variable, it’s a narrowband signal.
- Polyphase filters can be used to do this, but a broadband implementation requires many stages (high loss)
Weaver Architecture

\[ RF = m_r(t) \cos(\omega_{LO1} + \omega_{IF1})t + m_i(t) \cos(\omega_{LO1} - \omega_{IF1})t \]

\[ IF_1 = LO_1 - RF \]

\[ IF = LO_2 - IF_1 = LO_2 - LO_1 + RF = RF - (LO_1 - LO_2) \]

- Eliminates the need for a phase shift in the signal path. Easier to implement phase shift in the LO path.
- Can use a pair of quadrature VCOs. Requires 4X mixers!
- Sensitive to second image.

\[ A_{LPF} = \cos \omega_{LO1} t \times RF = \frac{m_r}{2} \cos(\omega_{IF1})t + \frac{m_i}{2} \cos(\omega_{IF1})t \]

\[ B_{LPF} = \sin \omega_{LO1} t \times RF = -\frac{m_r}{2} \sin(\omega_{IF1})t + \frac{m_i}{2} \sin(\omega_{IF1})t \]

\[ C_{LPF} = A \times \cos \omega_{LO2} t = \frac{m_r}{4} \cos(\omega_{IF})t + \frac{m_i}{4} \cos(\omega_{IF})t \]

\[ D_{LPF} = B \times \sin \omega_{LO2} t = -\frac{m_r}{4} \cos(\omega_{IF})t + \frac{m_i}{4} \cos(\omega_{IF})t \]

\[ IF = C - D = \frac{m_r}{2} \cos \omega_{IF} t \]
Direct Conversion Architecture
Direct Conversion (Zero-IF)

\[ \omega_{RF} = \omega_{LO} = \omega_0 \]

\[ m_r(t) \cos(\omega_{RF})t \times \cos(\omega_{LO})t = \frac{1}{2} m_r(t) (1 + \cos(2\omega_0)t) \]

- The most obvious choice of LO is the RF frequency, right? 
  \[ IF = LO - RF = DC? \]
- Why not?
- Even though the signal is its own image, if a complex modulation is used, the complex envelope is asymmetric and thus there is a “mangling” of the signal
Use orthogonal mixing to prevent signal folding and retain both I and Q for complex demodulation (e.g. QPSK or QAM).

Since the image and the signal are the same, the image-reject requirements are relaxed (it’s an SNR hit, so typically 20-25 dB is adequate).
Problems with Zero-IF

\[ LO = p(t) \cos(\omega_{LO} t + \phi(t)) \]

\[ LO \times LO = p(t)^2 + p(t)^2 \cos(2\omega_{LO} t + 2\phi(t)) \]

- Self-mixing of the LO signal is a big concern.
- LO self-mixing degrades the SNR. The signal that reflects from the antenna and is gained up appears at the input of the mixer and mixes down to DC.
- If the reflected signal varies in time, say due to a changing VSWR on the antenna, then the DC offset is time-varying.
DC offsets that appear at the baseband experience a large gain. This signal can easily saturate out the receive chain.

A large AC coupling capacitor or a programmable DC-offset cancellation loop is required. The HPF corner should be low (kHz), which requires a large capacitor.

Any transients require a large settling time as a result.
Sensitivity to 2\textsuperscript{nd} Order Disto

- Assume two jammers have a frequency separation of $\Delta f$:
  \begin{align*}
  s_1 &= m_1(t) \cos(\omega_1 t) \\
  s_2 &= m_2(t) \cos(\omega_1 t + \Delta \omega t)
  \end{align*}

  \[(s_1 + s_2)^2 = (m_1(t) \cos \omega_1 t)^2 + (m_2(t) \cos \omega_2 t)^2 + 2m_1(t)m_2(t) \cos(\omega_1 t) \cos(\omega_1 + \Delta \omega) t\]

  \[
  \text{LPF}\{(s_1 + s_2)^2\} = m_1(t)^2 + m_2(t)^2 + m_1(t)m_2(t) \cos(\Delta \omega) t
  \]

- The two produce distortion at DC. The modulation of the jammers gets doubled in bandwidth.
- If the jammers are close together, then their inter-modulation can also fall into the band of the receiver.
- Even if it is out of band, it may be large enough to saturate the receiver.
Sensitivity to $1/f$ Noise

- Since the IF is at DC, any low frequency noise, such as $1/f$ noise, is particularly harmful.
- CMOS has much higher $1/f$ noise, which requires careful device sizing to ensure good operation.
- Many cellular systems are narrowband and the entire baseband may fall into the $1/f$ regime!
- Example: GSM has a 200 kHz bandwidth. Suppose that the flicker corner frequency is 100 kHz. The in-band noise degradation is thus:

$$\bar{v}_{\text{ave}}^2 = \frac{1}{200\text{kHz}} \left[ \int_{1\text{kHz}}^{100\text{kHz}} \frac{a}{f} df + \int_{100\text{kHz}}^{200\text{kHz}} b df \right]$$

$$a = 1\text{kHz} \cdot 100 \cdot \bar{v}_i^2$$

$$b = \bar{v}_i^2$$

$$\bar{v}_{\text{ave}}^2 = \frac{1}{200\text{kHz}} \left[ a \ln \frac{100k}{1k} + b(200k - 100k) \right] = \frac{1}{200\text{kHz}} (11.5a + 100kb) = \frac{1}{200\text{kHz}} (11.5 \cdot 100 \cdot 100 \cdot \bar{v}_i^2 + 100 \cdot \bar{v}_i^2) = 6.25\bar{v}_i^2$$
DC offset and flicker noise a major concern. In addition to the dominant mechanisms described earlier, other source of DC offset include:
- Strong blocker signal can leak into mixers and self-mix
- Non 50% duty cycle of LO can cause output DC offset

Second order distortion a major concern and requires linear LNA/mixer.

Despite these limitations, the zero-IF has many advantages, such as elimination of the image rejection problem and the elimination of all external SAW filters. This has made it a very popular choice.
Low-IF and Double Conversion Architectures
Instead of going to DC, go a low IF, low enough so that the IF circuitry and filters can be implemented on-chip, yet high enough to avoid problems around DC (flicker noise, offsets, etc). Typical IF is twice the signal bandwidth.

The image is rejected through a complex filter. Polyphase filters are popular choices.
Designing a receiver with two separate frequency synthesizers (PLL’s) that are not integer related requires double the hardware and may suffer from frequency pulling (interaction). Derive second LO by dividing first:

\[ \omega_{RF} = \omega_1 + \omega_2 = \omega_1 \left(1 + \frac{1}{N}\right) \]

This architecture is the sliding IF receiver, so called because the IF frequency is not fixed but moves.
For $N = 2$ (“free” quadrature), we have $\omega_{RF} = \omega_1 \frac{3}{2}$, so that the first IF is given by

$$\omega_{IF} = \omega_{RF} - \omega_1 = \omega_{RF} \left(1 - \frac{2}{3}\right) = \frac{1}{3} \omega_{RF}$$

As the input frequency moves, the first IF “slides” along. Interestingly the image band is not as wide as the RF band. Since the IF scales as we move the input, the image band compresses:

$$\omega_{IM,1} = \omega_{RF,1} - 2(\omega_{RF,1} - \omega_{LO,1}) = \omega_{RF,1} \left(1 - 2(1 - \frac{2}{3})\right) = \frac{1}{3} \omega_{RF,1}$$

$$\omega_{IM,2} = \frac{1}{3} \omega_{RF,2}$$

$$B_{Image} = \omega_{IM,2} - \omega_{IM,1} = \frac{1}{3} B_{RF}$$
The dual-conversion double-quad architecture has the advantage of de-sensitizing the receiver gain and phase imbalance of the I and Q paths.
Assuming ideal quadrature and no gain errors:

\[ RF = m_r(t) \cos(\omega_{LO1} + \omega_{LO2} + \omega_{IF})t + m_i(t) \cos(\omega_{LO1} + \omega_{LO2} - \omega_{IF})t + \]

\[
A = \text{LPF}\{RF \times \cos(\omega_{LO1}t)\} = \frac{1}{2} \left\{ \begin{array}{c} m_r(t) \cos(\omega_{LO2} + \omega_{IF})t + \\
m_i(t) \cos(\omega_{LO2} - \omega_{IF})t \end{array} \right\}
\]

\[
B = \text{LPF}\{RF \times \sin(\omega_{LO1}t)\} = \frac{1}{2} \left\{ \begin{array}{c} -m_r(t) \sin(\omega_{LO2} + \omega_{IF})t \\
-m_i(t) \sin(\omega_{LO2} - \omega_{IF})t \end{array} \right\}
\]

\[
C = \text{LPF}\{A \times \cos(\omega_{LO2}t)\} = \frac{1}{2} \left\{ \begin{array}{c} m_r(t) \cos(\omega_{IF})t + \\
m_i(t) \cos(\omega_{IF})t \end{array} \right\}
\]

\[
D = \text{LPF}\{A \times \sin(\omega_{LO2}t)\} = \frac{1}{2} \left\{ \begin{array}{c} m_r(t) \sin(\omega_{IF})t + \\
-m_i(t) \sin(\omega_{IF})t \end{array} \right\}
\]

\[
E = \text{LPF}\{B \times \cos(\omega_{LO2}t)\} = \frac{1}{2} \left\{ \begin{array}{c} m_r(t) \sin(\omega_{IF})t + \\
-m_i(t) \sin(\omega_{IF})t \end{array} \right\}
\]

\[
F = \text{LPF}\{B \times \sin(\omega_{LO2}t)\} = \frac{1}{2} \left\{ \begin{array}{c} -m_r(t) \cos(\omega_{IF})t + \\
-m_i(t) \cos(\omega_{IF})t \end{array} \right\}
\]

\[
I = C - F = (m_r(t) + m_i(t)) \cos(\omega_{IF})t
\]

\[
Q = D + E = (m_r(t) - m_i(t)) \sin(\omega_{IF})t
\]
Gain Error Analysis

\[ RF = m_r(t) \cos(\omega_{LO1} + \omega_{LO2} + \omega_{IF})t + m_i(t) \cos(\omega_{LO1} + \omega_{LO2} - \omega_{IF})t + \]

\[ A = \text{LPF}\{RF \times \left(1 + \frac{\Delta a_1}{2}\right) \cos(\omega_{LO1}t)\} = \frac{1}{2} \left(1 + \frac{\Delta a_1}{2}\right) \left\{ m_r(t) \cos(\omega_{LO2} + \omega_{IF})t + m_i(t) \cos(\omega_{LO2} - \omega_{IF})t \right\} \]

\[ B = \text{LPF}\{RF \times \left(1 - \frac{\Delta a_1}{2}\right) \sin(\omega_{LO1}t)\} = \frac{1}{2} \left(1 - \frac{\Delta a_1}{2}\right) \left\{ -m_r(t) \sin(\omega_{LO2} + \omega_{IF})t - m_i(t) \sin(\omega_{LO2} - \omega_{IF})t \right\} \]

\[ C = \text{LPF}\{A \times \left(1 + \frac{\Delta a_2}{2}\right) \cos(\omega_{LO2}t)\} = \frac{1}{2} \left(1 + \frac{\Delta a_1}{2}\right) \left(1 + \frac{\Delta a_2}{2}\right) \left\{ m_r(t) \cos(\omega_{IF})t + m_i(t) \cos(\omega_{IF})t \right\} \]

\[ D = \text{LPF}\{A \times \left(1 - \frac{\Delta a_2}{2}\right) \sin(\omega_{LO2}t)\} = \frac{1}{2} \left(1 + \frac{\Delta a_1}{2}\right) \left(1 - \frac{\Delta a_2}{2}\right) \left\{ m_r(t) \sin(\omega_{IF})t + m_i(t) \sin(\omega_{IF})t \right\} \]

\[ E = \text{LPF}\{B \times \left(1 + \frac{\Delta a_2}{2}\right) \cos(\omega_{LO2}t)\} = \frac{1}{2} \left(1 - \frac{\Delta a_1}{2}\right) \left(1 + \frac{\Delta a_2}{2}\right) \left\{ m_r(t) \sin(\omega_{IF})t + m_i(t) \sin(\omega_{IF})t \right\} \]

\[ F = \text{LPF}\{B \times \left(1 - \frac{\Delta a_2}{2}\right) \sin(\omega_{LO2}t)\} = \frac{1}{2} \left(1 - \frac{\Delta a_1}{2}\right) \left(1 - \frac{\Delta a_2}{2}\right) \left\{ -m_r(t) \cos(\omega_{IF})t - m_i(t) \cos(\omega_{IF})t \right\} \]

\[ I = C - F = (1 + \Delta a_1 \Delta a_2)(m_r(t) + m_i(t)) \cos(\omega_{IF})t \]

\[ Q = D + E = (1 - \Delta a_1 \Delta a_2)(m_r(t) - m_i(t)) \sin(\omega_{IF})t \]

- The gain mismatch is reduced due to the product of two small numbers (amplitude errors).
Phase Error Analysis

\[
RF = m_r(t) \cos(\omega_{LO1} + \omega_{LO2} + \omega_{IF})t + m_i(t) \cos(\omega_{LO1} + \omega_{LO2} - \omega_{IF})t +
\]

\[
A = \text{LPF}\{RF \times \cos(\omega_{LO1} t + \phi_1)\} = \frac{1}{2} \begin{cases} 
  m_r(t) \cos(\omega_{LO2} + \omega_{IF} + \phi_1)t + \\
  m_i(t) \cos(\omega_{LO2} - \omega_{IF} + \phi_1)t 
\end{cases}
\]

\[
B = \text{LPF}\{RF \times \sin(\omega_{LO1} t - \phi_1)\} = \frac{1}{2} \begin{cases} 
  -m_r(t) \sin(\omega_{LO2} + \omega_{IF} - \phi_1)t + \\
  -m_i(t) \sin(\omega_{LO2} - \omega_{IF} + \phi_1)t 
\end{cases}
\]

\[
C = \text{LPF}\{A \times \cos(\omega_{LO2} t + \phi_2)\} = \frac{1}{2} \begin{cases} 
  m_r(t) \cos(\omega_{IF} + \phi_1 + \phi_2)t + \\
  m_i(t) \cos(\omega_{IF} + \phi_1 + \phi_2)t 
\end{cases}
\]

\[
D = \text{LPF}\{A \times \sin(\omega_{LO2} t - \phi_2)\} = \frac{1}{2} \begin{cases} 
  m_r(t) \sin(\omega_{IF} - \phi_1 - \phi_2)t + \\
  -m_i(t) \sin(\omega_{IF} - \phi_1 - \phi_2)t 
\end{cases}
\]

\[
E = \text{LPF}\{B \times \cos(\omega_{LO2} t + \phi_2)\} = \frac{1}{2} \begin{cases} 
  m_r(t) \sin(\omega_{IF} + \phi_1 + \phi_2)t + \\
  -m_i(t) \sin(\omega_{IF} + \phi_1 + \phi_2)t 
\end{cases}
\]

\[
F = \text{LPF}\{B \times \sin(\omega_{LO2} t - \phi_2)\} = \frac{1}{2} \begin{cases} 
  -m_r(t) \cos(\omega_{IF} - \phi_1 - \phi_2)t + \\
  -m_i(t) \cos(\omega_{IF} - \phi_1 - \phi_2)t 
\end{cases}
\]

\[
l = C - F = (m_r(t) + m_i(t)) \cos(\phi_1 + \phi_2) \cos(\omega_{IF} t)
\]

\[
Q = D + E = (m_r(t) - m_i(t)) \cos(\phi_1 + \phi_2) \sin(\omega_{IF} t)
\]

\[
\cos(\phi_1 + \phi_2) \approx \left(1 - \frac{(\phi_1 + \phi_2)^2}{2}\right)
\]

- The phase error impacts the I/Q channels in the same way, and as long as the phase errors are small, it has a minimal impact on the gain of the I/Q channels.
Essentially a complex mixer topology. Mix RF I/Q with LO I/Q to form baseband I/Q

Improved image rejection due to desensitization to quadrature gain and phase error.
References


